Polyrhythmic Configurations, Structures, and Tempo

A Self-Observational Study of Single-Line Polyrhythms in Percussion Performance

Viktor Fröjd

Master of Music in Music Theory

Spring semester 2022

Supervisor: Associate Professor Laura Gorbe-Ferrer

Norges musikkhøgskole
Norwegian Academy of Music

Acknowledgements

I want to thank my supervisor Dr. Laura Gorbe-Ferrer for kind support and valuable suggestions throughout this past year.

I thank Deborah Longenecker for helping me organise the structure of the thesis and significantly improve my use of the English language.

I also thank Rúben Carvalho for suggesting improvements on clarity and finding errors I otherwise would have missed.

Finally, I thank Markus Tähti Dunseth for translating my abstract into Norwegian.

Abstract

Although polyrhythms are usually defined as two or more rhythms of incongruent subdivisions, polyrhythms can also be performed as single-line polyrhythms, in which the played rhythm is incongruent with an experienced pulse. This thesis explores the performance possibilities of single-line polyrhythms in a practice-led, self-observational study, which investigates how polyrhythmic configurations relate to tempo limits and how tempo influences polyrhythmic structure (i.e. mental organisation) in the context of percussion performance. For the study, I designed a large set of rhythmic exercises, informed by a theoretical exploration of polyrhythmic types and pedagogical approaches; I then played these exercises in a wide range of tempos and took notes on my experiences during the practice sessions. The results show that the polyrhythmic configurations in the exercises influenced the playable tempo range for each exercise, with differences shown between fast and slow tempo ranges. In addition, the results show that various performance strategies, such as mental subdivision or motion-based strategies, can facilitate performance for certain tempo ranges and polyrhythmic configurations.

Sammendrag

Norsk tittel: *Polyrytmiske mønstre, strukturer og tempo: En selvobserverende studie om enstemmige polyrytmer i slagverkskontekst*

Selv om polyrytmer ofte defineres som to eller flere rytmer med kontrasterende underdelinger kan polyrytmer også spilles som enstemmige mønster, der den spilte rytmen står i kontrast til den opplevde pulsen. Denne masteroppaven utforsker mulighetene i fremføring av slike enstemmige polyrytmer gjennom en praksisledet, selvobserverende studie. Med utgangspunkt i fremføring på slagverk undersøker studien hvordan spillbarheten til polyrytmiske strukturer forholder seg til tempo, og hvordan tempo også påvirker polyrytmens struktur. Til studien skrev jeg en rekke polyrytmiske etyder, med grunnlag i en teoretisk diskusjon av ulike typer polyrytmikk og pedagogiske tilnærminger. Deretter spilte jeg etydene i et bredt spenn av tempi, og noterte erfaringer fra øvingen. Resultatene viser at de polyrytmiske mønstrene i etydene påvirket hvilke tempi etydene kunne spilles i, med forskjeller mellom raske og langsomme tempi. I tillegg viser resultatene at ulike spillestrategier, for eksempel mental underdeling og bevegelsesorienterte strategier kan legge til rette for spill i visse tempi og av polyrytmiske mønstre.

Table of Contents

List of Tables

List of Figures

1. Introduction

1.1 Background

Whether found in the juxtapositions in Cage's *Third Construction*, the syncopated patterns in Sandström's *Drums*, or the extreme nested rhythms of Ferneyhough's *Bone Alphabet*, polyrhythms provide a nuanced rhythmic texture in music but can be challenging to perform. In the literature, polyrhythms are commonly described solely as a mathematical ratio (i.e. 3:2 or 5:4), and pedagogical methods often take ratio-based approaches to teaching polyrhythms (Chaffee, 1976; Holmqvist, 2010; Jersild, 1975; Magadini, 1993; Palmqvist, 2006). However, prioritising the abstract ratio of a polyrhythm over the performer's experience of the polyrhythm may in fact make its rhythmic structure seem more complex than it actually is for the performer (Cook, 1999, pp. 258–259). Thus, there is a need for a more nuanced understanding of the concept of polyrhythms that acknowledges and prioritises the experiences of the performer. My personal experience as a percussionist, combined with my academic interest in the music theory of polyrhythms, influences the direction of this study, which aims to explore performance possibilities of single-line polyrhythms in a wide range of polyrhythmic configurations and tempos. This introduction will define key concepts essential to the theoretical basis of the study, review the current research literature on polyrhythmic perception and performance, present the aim of the study and its research questions, and discuss the methodology of the study.

1.2 Key Concepts and Literature Review

In the following section, I make a distinction between *polyrhythmic configuration*, which refers to abstract ratio of the polyrhythm, and *polyrhythmic structure*, which refers to the polyrhythm as performed. I further distinguish between different forms of polyrhythmic performance. I then give an overview of perceptual thresholds relating to rhythm and research literature on polyrhythmic perception and performance.

1.2.1 Polyrhythmic Configuration and Polyrhythmic Structure

To illustrate how polyrhythms are more than simply an abstract ratio, I will examine two passages from Gordon Stout's *Two Mexican Dances for Marimba, No. 2*, which I studied and performed during my pre-college studies.

In measures 64–69 (Figure 1), the polyrhythms have an uneven grouping structure and a free, rubato character, leaving much room for interpretation and flexible timing. In contrast, the polyrhythms in measures 42–44 (Figure 2) are evenly divided and have a steady, groovy character and should be played with drive and precision.

Figure 1

Stout (1977), Two Mexican Dances for Marimba, No. 2*, measures 64–69.*

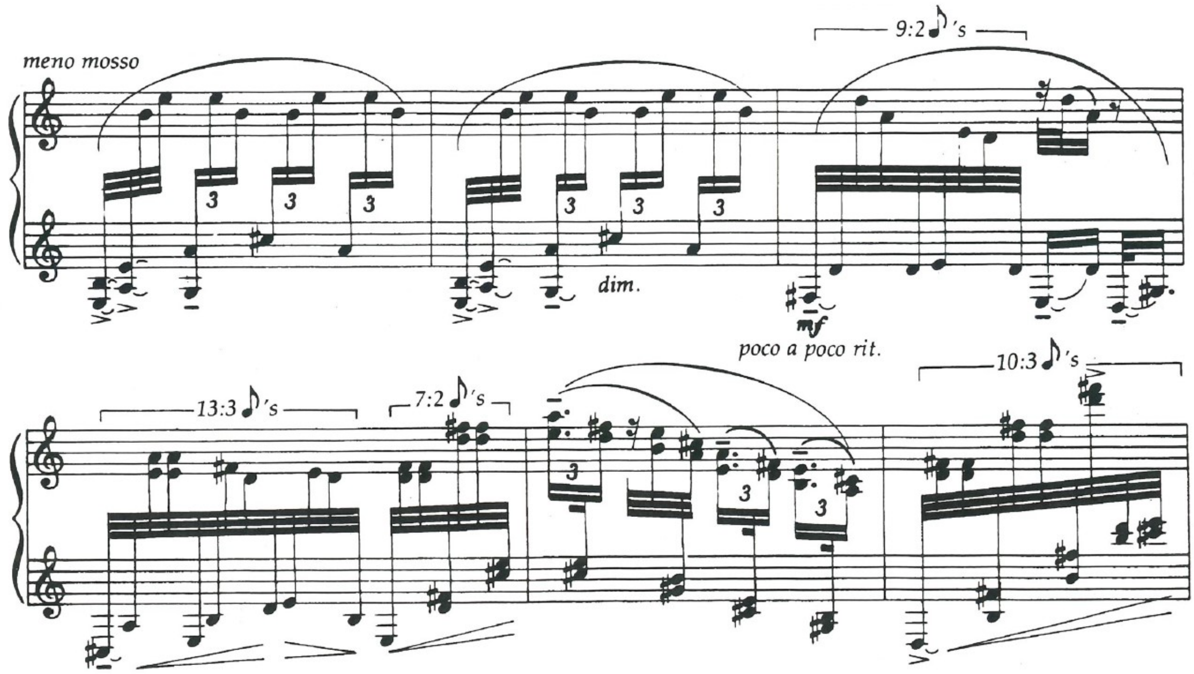


Figure 2

Stout (1977), Two Mexican Dances for Marimba No. 2*, measures 42–44.*

When I first played this piece, I did not pause to consider the underlying reasons for the difference between the rhythms in the two passages. Although the rhythms have different structures, notations, and expressive qualities, the rhythms in measure 66 (Figure

1) and measures 43 and 44 (Figure 2) both have a rhythmic ratio of 9:2. The grouping structure of the 9:2 rhythm in measures 43 and 44 has a hierarchical organisation in which sixteenth-note triplets are embedded within a framing eighth-note triplet, whereas the grouping structure of the 9:2 rhythm in measure 66 might be best understood as a 4+5 grouping. In a practice situation, the rhythm in Figure 1 can be practiced by first learning the framing triplet rhythm and later adding the embedded subdivisions. In contrast, the rhythm in Figure 2 could be practiced by first approximating the polyrhythm and trying to smooth it out over the time-span.

This example shows that two polyrhythms with the same rhythmic ratio might be structured in different ways and, in practice and performance, are often approached in different manners. Instead of solely defining polyrhythms by an abstract ratio, this distinction acknowledges that rhythmic structure is not predetermined in music but is shaped by the performer; in John Rink's words, "Musical materials do not in themselves constitute structure(s): they *afford* the inference of structural relationships" (2015, p. 129, emphasis in original). To make a distinction between the polyrhythm as abstract ratio and polyrhythms as performed rhythms, I will refer to the abstract ratio of polyrhythms as *polyrhythmic configuration* and the mental organisation of polyrhythms in performance as *polyrhythmic structure*.[1]

Furthermore, the examples in Figures 1 and 2 show that polyrhythms are not limited to "two or more separate rhythmic streams in the musical texture whose periodicities are noninteger multiples" (London, 2012, p. 66), as in the usual definition of polyrhythms, but can also exist in the form of what I term a *single-line polyrhythm* – a rhythm that is incongruent with a pulse that is experienced but not played. In addition, polyrhythms are not limited to solo performances but can be collaboratively performed by multiple members of an ensemble. Single-line polyrhythms and collaborative polyrhythms are rarely considered in polyrhythm studies[2] but are reflected in pedagogical literature, where commonly suggested practice methods include singing the polyrhythm while clapping or conducting the pulse, practicing with a metronome, and ensemble practice of polyrhythms (Jersild, 1975; Magadini, 1993; Palmqvist, 2006). In this way, pedagogical literature tends to acknowledge a greater variety of forms of polyrhythms than research literature does.

---

[1] The term configuration is borrowed from Handel (1984).
[2] Bogacz (2005) and Grieshaber (1990) are notable exceptions.

1.2.2 The Influence of Tempo

Not only can polyrhythmic grouping structure contribute to differences between polyrhythms of the same abstract ratio, but performance tempo shapes the polyrhythms by exerting a strong influence on rhythmic character. For example, although the rhythms of the opening fanfare in the last section of Rossini's *William Tell Overture* (Figure 3) and the opening of the second movement in Beethoven's *Symphony No. 7* (Figure 4) are remarkably similar (aside from notational differences), their rhythmic character is markedly different, largely due to the difference in tempo.

Figure 3

Rossini (1829/1994), William Tell Overture*, measures 234–239 (notation edited for clarity).*

Figure 4

Beethoven (1816/1863), Symphony No. 7*, 2nd movement, measures 1–10.*

The different characters of the rhythms in the Beethoven and Rossini examples are shaped by thresholds of rhythm perception (i.e. perceptual thresholds), which also influence the performance of polyrhythms. Following is a short review of the perceptual thresholds concerning rhythm and tempo, as well as a review of published polyrhythm studies and their relation to tempo.

Imagine a computer-generated rhythm, which can be sped up or slowed down at will. In moderate tempos one can easily follow and tap along to the rhythm, as long as the rhythm is fairly simple. If the rhythm is slowed down, it will at some point lose its feeling of motion and coherence (Gabrielsson, 1993, p. 97). Likewise, if the rhythm is sped up, it will at some point become too fast to follow. These thresholds depend on cognitive and perceptual constraints, which vary slightly from person to person. The fastest tempo in which successive notes can be perceived as rhythms is about 600bpm, or a 100ms inter-onset interval (London, 2012, p. 27).[3] In faster tempos the notes become blurred and are perceived as a grace note or a trill. On the other end, the slowest tempo possible to follow is around 30bpm or two seconds (London, 2012, p. 30). Below this tempo the sense of pulse is lost, and consecutive events are perceptually separated. Related to the perceptual threshold of slow tempos is the psychological present, which usually extends to about two seconds but can extend to four to six seconds (London, 2012, p. 30). In terms of rhythms, the psychological present can be defined as the time-span in which one can hear a rhythm as a spontaneously perceived whole (Gabrielsson, 1993, p. 97).

These perceptual thresholds describe a general frame for rhythm perception – or, in London's (2012, p. 41) term, a "metric envelope". The thresholds constrain the metrical possibilities in a given tempo: subdivisions are limited in high tempos, whereas the hierarchical depth of measures (and "hypermeasures") is limited in slow tempos (London, 2012, p. 47). Within the metric envelope, qualitative differences affect perception; not all tempos are perceived in the same way. Paul Fraisse (as cited in Clarke, 1999, p. 474) made a distinction between *temps courts* (short times) for tempos around 150bpm (400ms) or higher, and *temps longes* (long times) for tempos lower than 150bpm – with some margin for transition between the two categories. In *temps courts* rhythmic events tend to group,

[3] Many studies report durations in milliseconds. In a musical context, beats per minutes (bpm) is a more familiar measurement and will in most cases be used to refer to these durations. Bpm can be turned into milliseconds (ms) by the formula (1/bpm)*60000=ms; milliseconds (ms) can be turned into bpm by switching the positions of "bpm" and "ms" in the same formula. A conversion table is included in Appendix 1, Table A.

whereas in *temps longes* rhythmic events have duration proper.[4] According to London (2002, pp. 538–540), an upper limit for pulse tempo is set by a threshold at 240bpm (250ms), above which events are judged holistically as subdivisions. Finally, tempos around 100bpm (600ms) have the special quality of being neither too fast nor too slow, a sort of mean tempo in which pulse perception is most salient (London, 2012, p. 31).

Rhythms are often heard and performed in a metrical context. Since meter is a multileveled mental construct, a mode of attending, various levels in the metrical hierarchy (e.g. subdivisions, beats, and measures), will fall into different qualitative categories of time (London, 2012). This qualitative categorisation may shape the perception of rhythms, including polyrhythms, into a hierarchical gestalt pattern, in which the sense of tempo is a sum effect of the interaction of all present levels (Handel, 1984, p. 483; London, 2012, p. 33). However, perceptual thresholds do not dictate strict limits in real-life performance. To give an example, Polak (2018, p. 220) reports drummers playing as fast as 750bpm (80ms) in recordings of West African dance music, which suggests that real-life music making "facilitates faster metric subdivisions than the dead-pan isochronous equitones used in laboratory experiments".

Studies seem to indicate that polyrhythms are playable in a narrower tempo range than the more general metric envelope. Fraisse (1982, as cited in Grieshaber, 1990, p. 13) suggests that polyrhythms become unstable when the full cycle of the rhythm exceeds 1580ms (38bpm). Handel (1984, p. 473) found that pulse trains in polyrhythms were difficult to follow in note rates slower than 75bpm, or faster than 300bpm, but could be followed in 40bpm and 500bpm under some conditions.[5] This result suggests that note duration, rather than the length of the cycle, constrains the perception of polyrhythms. However, such studies often are based on tapping tasks and listening tasks using telegraph-like equipment, far removed from genuine musical situations (e.g. Beauvillain & Fraisse, 1984; Klapp et al., 1998).

In a more realistic performance situation, Bogacz (2005) studied the 5:3 polyrhythm with trained pianists playing on a keyboard. With the possibility of using all five fingers, the

[4] Specific numerical values for Fraisse's durations vary between sources, but the qualitative difference is more important than the specific durations. (Compare Clarke, 1999, pp. 474–475; Handel, 1989, p. 402; London, 2002, pp. 539–541)
[5] In the article reported as 800ms, 200ms, 1500ms, and 120ms durations, respectively.

pianists were able to play the polyrhythm in both very high and low tempos ranging from one note per second to sixteen notes per second.[6] In the third experiment of the study, the polyrhythm was split between two pianists, producing similar results, suggesting that these tempos are likely close to what can be expected of performers in an ensemble situation or when playing a single-line polyrhythm.

An important finding in Bogacz's (2005) study is that the same polyrhythm activates the motor system differently depending on performance speed, shifting from a one-to-one control of the notes in slow tempos to a more global control of the movement in fast tempos. The tempo at which the shift occurs corresponds quite closely to the 240bpm threshold of beat perception.[7] Similarly, Clarke (1999, p. 495) notes that the individual notes in subdivisions of the beat are not directly timed, but their timing is shaped by the physical motion involved in playing them. A study by Mclaughlin & Boals (2010) suggests that even if the notes are played in the same tempo, the mental organisation of a rhythm (in this case, whether the pulse is perceived as fast or slow) affects how accurately and quickly the rhythm can be played. These findings suggest that the mental organisation of the rhythm (i.e. the rhythmic structure) and the physical motions involved in playing are affected by tempo range. One can therefore expect a mental organisation or strategy that might work well for slow tempos to be less effective – if even possible – in higher tempos, and vice versa.

Regarding the relationship between polyrhythmic configuration and tempo, Grieshaber (1990) found that the complexity of the polyrhythmic patterns clearly influenced performance accuracy. Performers played the 3:2 rhythm more accurately than 4:3, which in turn was better than 5:4 (pp. 173–174). Grieshaber suggests that the increasing difficulty might be due to increasing complexity, measured as number of elements per rhythm. However, it could also be explained by the increasing length of the rhythmic cycle, since the faster part of the polyrhythm was played at the same speed for all configurations (pp. 179–180).

[6] The slowest tempo corresponds to a cycle length of seven seconds. The "fives" were 1400ms long (43bpm) and the "threes" 2333ms long (26bpm). The fastest corresponds to a cycle length of 437ms or 137bpm. The "fives" were 88ms long (686bpm) and the "threes" were 146ms long (411bpm) (Bogacz, 2005, p. 25).
[7] Around six notes per second, in which the note rate of the "fives" was 257bpm.

Supporting the argument that the length of the rhythmic cycle affects the stability of a polyrhythm, Bogacz (2005) tested a single polyrhythm in a wide range of tempos and found that performance was most unstable in slow tempos. Since performers are constrained by perceptual thresholds, and more complex polyrhythms necessarily require a longer span of time when played at a constant note rate, it seems that more complex polyrhythms are constrained by tempo to a higher degree.

However, there are some significant gaps in the research. Although the complexity of a polyrhythmic configuration seems to influence its performable tempo range, polyrhythm studies have only tested a limited number of polyrhythms, and it is not clear if more complex configurations than those tested are increasingly limited by tempo. Handel (1984) tested listener responses to combinations of 2-, 3-, 4-, 5-, and 7-note cycles, which seems to be one of the most extensive studies in this regard. In actual musical works, complex polyrhythms are frequent, notably in the music of the so-called New Complexity, represented by composers such as Brian Ferneyhough and Michael Finnissy (Bortz, 2003). The existence of such music shows that more complex polyrhythms than those most often studied are possible to play. Thus, there is a need to explore a wider range of polyrhythms in a performance context.

In addition, since not all tempos are perceived in the same way, it is likely that certain strategies for playing polyrhythms might be limited to certain tempo ranges. While the pedagogical literature contains some remarks on this topic (e.g. Chaffee, 1976; Informance, 2019; Jersild, 1975), studies focusing on performance strategies for polyrhythms in various tempos do not seem to exist. Grieshaber's (1990) study comes the closest, reporting that the participants engaged in various strategies for coping with polyrhythms that corresponded to strategies used in pedagogical literature; however, Grieshaber's study only tested the polyrhythms in a single tempo.

Moreover, polyrhythmic performance studies have often been limited to two-handed playing; other forms of polyrhythmic playing might not be constrained in the same way as two-handed playing. One could expect single-line polyrhythms, such as those in Stout's *Two Mexican Dances for Marimba No. 2*, to allow a wide range of tempos to be playable, since the hands are working together and not against each other, and to allow for more complex polyrhythmic configurations. Despite the existence of much percussion repertoire that features single-line polyrhythms, the issue of tempo and complexity has not been thoroughly explored in single-line polyrhythmic performance.

1.3 Aim and Research Questions

Studies on polyrhythmic performance suggest that a polyrhythm with a more complex configuration might be more limited by tempo than a polyrhythm with a less complex configuration. Within the performable tempo range for a polyrhythm, different polyrhythmic structures (i.e. different mental organisations) influence how the polyrhythm is experienced and performed. However, it is not clear if more complex polyrhythms are similarly constrained in single-line polyrhythmic performance, and neither is it clear how the experience and performance strategies for a polyrhythm differ in various tempos. Therefore, this thesis aims to explore performance possibilities of single-line polyrhythms in a wide range of polyrhythmic configurations, with a special consideration for how different tempos influence polyrhythmic structure in the context of percussion performance. To explore these issues, the following questions are posed:

1. How does the polyrhythmic configuration (i.e. the abstract ratio) relate to tempo limits for single-line polyrhythms in percussion performance?
2. How do different tempos influence the polyrhythmic structure (i.e. the mental organisation) for single-line polyrhythms in percussion performance?

While designing the study to explore these questions, I expected to find more complex polyrhythmic exercises to be playable in a narrower tempo range than less complex polyrhythmic exercises. I also expected that longer cycles and more complex relations between the played rhythm and the pulse would make performance more difficult in slow tempos, with similar constraints in fast tempos. In addition, since the exercises in the practical study consisted of two rhythms per exercise, I expected the relation between the two rhythms to affect the playable tempo range. Finally, regarding polyrhythmic structure, I assumed that the possible ways of mentally structuring the polyrhythms could be influenced by the configurations of the exercise and by the played tempo.[8]

[8] Key terms used in the practical study are described in sections 2.4, 3.1, and 3.2.

1.4 Outline of the Thesis

The thesis is divided into a theoretical discussion and a practical study.

Chapter 2 contains a theoretical discussion of polyrhythmic structures and pedagogical approaches to polyrhythms. The theoretical discussion informs the design of the exercises used in the practical study and situates the practical study in a musical context.

Chapter 3 presents, in detail, the design of the exercises, how the practice sessions were conducted, and how data was collected and analysed. The exercises were designed to include many aspects which might influence performance of polyrhythms and were played in a wide range of tempos. Playable tempo ranges and experiences were noted during each session.

Chapter 4 presents the results from the practice sessions, including both tempo limits for the configurations and possible rhythmic structures in various tempo ranges. Regarding tempo limits, the results are presented in sets of exercises, analysed for tendencies across exercises. Regarding rhythmic structures, the results include commonalities found across exercises, as well as cases of special interest.

Chapter 5 includes a discussion of the results and conclusions. The results show that the configurations influence the playable tempo range, with different conditions for fast and slow tempos. Different polyrhythmic structures were specific to certain tempo ranges, and certain performance strategies could facilitate performance for some tempos and configurations.

1.5 Methodology

1.5.1 Choice of Method

In the following section, I explain and justify my choice of method for this study. In addition, I discuss the understanding of knowledge in a performance context, problems regarding the measurement of complexity, the use of quantitative data, and the design of the practical study.

Since single-line polyrhythms have not been comprehensively studied, and the relation between polyrhythmic configurations and tempo limits is likely task-specific, predictions of the exact relationship between polyrhythmic configurations and tempo limits were difficult to draw. With little grounding for making assumptions, I instead took an explorative approach in a practice-led, self-observational study.

For testing tempo limits, a self-observational approach was a compromise solution. The data obtained was primarily quantitative, which normally would require a group of participants and the use of statistical methods. However, the practical matters associated with recruiting a group of sufficiently proficient musicians would have imposed substantial limitations on the number of exercises that could be tested in the study and require time and resources beyond the scope of this thesis. Because of these limitations, the quantitative results of the study were instead qualified by personal experiences from the practice sessions, and a sample of exercises were replayed to test the reliability of the results. Although including more participants would have been advantageous for testing specific hypotheses and would have produced more generalisable results, an explorative approach was better suited for a field where prior assumptions were at best speculative.

On the other hand, a self-observational approach was beneficial for studying the influence of tempo on polyrhythmic structure, since I was able to obtain first-hand experience, through practice, over an extended period of time. As Burke & Onsman (2017, p. 7) argue, research in artistic fields seeks to reveal the knowledge embodied in the processes involved in music making, which requires personal involvement and a self-reflective approach. Since musical knowledge is primarily gained through practice, practicing can sometimes be the only viable research method for exploring performance (Redgate, 2015, p. 214). While the self-observational quality of the study limits the transferability of the results to other contexts, the benefit of deeper insight in an unexplored area outweighed these drawbacks.

The practical study was designed with the aim to explore polyrhythmic performance from a musician’s perspective. While the design of the study has several experiment-like features, it is not an experiment; I was not an objective observer, and the study was not designed to test specific hypotheses, nor to produce generalisable results. Even though I strived to have similar conditions for each practice session, the experiences that appeared during these sessions influenced the results and formed an important part of the qualitative data. Instead, I chose an experiment-like design because it allowed systematic exploration of how various aspects in the polyrhythmic configurations afforded the use of certain structures, strategies, and physical motions throughout the playable tempo range. The quantitative results were used to link experiences and strategies to specific tempos. Likewise, the tempo limits were informed by the qualitative data of personal experience, which identified aspects of performance that limited the playable range. The study thus connected qualitative aspects to the quantitative results.

Because of the heavy reliance on self-observation in this study, the findings of this study cannot be generalised beyond the context of this thesis but can provide valuable insights to form the basis for further investigation. My study is essentially artistic, but the focus is on technique rather than creative artistic processes. It could be categorised as "research for the arts", since "art is not so much the object of investigation, but its objective" (Borgdorff, 2006, p. 6). My aim is to provide tools and knowledge for musicians, teachers, and researchers to explore further.

1.5.2 Relevant Elements of Personal Background and Percussion Performance

In order to identify personal and instrumental factors that influence the study, I draw from elements mentioned in John Rink's principles for performance analysis, discussed in his essay *The (f)utility of performance analysis* (2015). Rink's principles emphasise that musical understanding is strongly context bound and that musical structure is not found in the score, nor in any single aspect of music making, but is rather shaped through the interaction of the elements present in the performance. These elements include the following:

- The individual performer, with all prior experience and knowledge she may have
- The instrument, with a range of idiosyncratic motions and sounds
- The musical material, "the score", with all its structural potentials and affordances

As the act of shaping musical structure is inextricably connected to all three aspects, each aspect bringing into play certain affordances and constraints, their individual contributions must be acknowledged to understand their interaction and to define the performative context.

The following section briefly states factors relating to each aspect which might influence the study. First, as I will be the musician playing, I provide an introduction to my personal background as a percussionist. Secondly, regarding the instrumental element, I define several characteristics of percussion practice and performance and how these are managed in the practical study. Finally, the musical material of this study – i.e. the polyrhythmic configurations – is discussed in Chapter 2.

I am a right-handed percussionist, born in 1994 in Sweden, and am 28 years old at the time of writing. I took my bachelor's degree in classical percussion at the Academy of Music and Drama, Gothenburg University, finishing my studies in 2020. During my first year in Gothenburg, my left arm was injured by extensive practice, which influenced the rest of my studies. Because of this injury, I took a break from performance courses in my third year and focused instead on theoretical studies, specifically on polyrhythms. My efforts resulted in a heavily theoretical bachelor's thesis, in which I outlined a theoretical systematisation of polyrhythms (Fröjd, 2019). In my bachelor's thesis, I developed the X^n:Y notation used in the practical section of this thesis; these theoretical studies were accompanied by extensive polyrhythmic practice. Since that time, I have continued my studies in organising, understanding, and playing polyrhythms. In addition, I have played much repertoire containing polyrhythmic passages; many of these pieces are used as examples in the theoretical discussion in Chapter 2.

Along with my personal background, the characteristics of my instrument – percussion – also influence the study. The two fundamental aspects of percussion performance can be summarised as "*the stroke* and that which is being struck (skin, metal, wood, etc.)" (Stene, 2016, p. 11, emphasis in original). These aspects are interrelated, yet they are not equal. Percussion playing almost ubiquitously involve "handedness" – i.e. "something that is 'done with the hands'" (Stuart, 2009, p. 48) – but it can seem that "percussionists have so many instruments that, in effect, they have none to with which the [sic] can genuinely identify" (Stene, 2016, p. 1). What is being struck is thus less defined than the manner of striking. In other words, percussion playing involves "no instrument, just sticks" (Schick, 2006, p. 3).

Nevertheless, the instrument played – whether a snare drum, a marimba, pieces of sandpaper, or a collection of rocks – shapes what can be played, how it is played, and even the performer's understanding of what is played (Doğantan-Dack, 2015, pp. 172–173). The question is then what can be considered a representative instrument for a practice distinguished for its lack of a representative instrument.

For the purposes of this study, which requires practicing polyrhythms without complicating the issue of instrumentation, I chose a simple solution: the snare drum, which is widely used for practicing general percussion technique in the Western tradition. The snare drum is characterised by a short, clear, and sharp attack, with a wide dynamic range. Tuned properly, it offers a lot of rebound, facilitating fast playing. For practical reasons, the exercises in the study were not played on the same snare drum, but I ensured that each snare drum I used had a comfortable rebound. Many exercises were played on my Wincent Slim Pad, a practice pad with good rebound and a soft but clear attack.

The choice of drum sticks also influences snare drum performance; for instance, long and heavy sticks give a powerful sound and a nice resistance in playing but can lack nuance and be straining to play with in quick tempos. I played all exercises with the same pair of Vic Firth SD2 Bolero sticks due to their pleasant balance and medium-light weight.

Lastly, the sticking pattern and the choice of leading hand (i.e. the hand that plays on the beat) can affect performance. To keep it simple, all exercises were practiced with an alternating sticking pattern. Generally, I led with both right and left hand, but for some exercises, leading with the left hand proved more difficult than leading with the right hand. In these cases, I usually switched to leading solely with the right hand so that the results were not lowered because of technical deficiency in the left hand.

1.5.3 Issues of Measurement

Although polyrhythmic complexity might potentially correspond to a more limited tempo range, defining polyrhythmic complexity itself is not straightforward. One measure of complexity regards the number of elements in the polyrhythm (e.g. 5:3 has eight elements), another the size of the least common multiple (5:3 would give 5*3=15) (Grieshaber, 1990, pp. 21–22), but as Toussaint (2013, pp. 109–110) points out, "there is no logical *a priori* reason why a mathematical measure of complexity should agree with either cognitive or performance complexity".

Instead of testing if a measure of theoretical complexity corresponded to performance complexity, it seemed more appropriate to take an open approach, in which performance complexity could be compared to various aspects in the polyrhythmic configurations, searching for commonalities across exercises. The aim was therefore to find experientially grounded measures of complexity rather than testing previously conceived theoretical measures of complexity.

Performance complexity can be reasonably equated with performance difficulty. In polyrhythmic studies, increased difficulty is frequently related to timing deviations; higher timing deviations indicate more difficult rhythms or a more difficult tempo (Bogacz, 2005, p. 25; Grieshaber, 1990, pp. 21–22). However, mechanical timing does not always correspond to artistic performance; timing patterns have been found to change with tempo for pianists playing scales, with higher variation in high and low tempos (Bengtsson, 1987, p. 78; Gabrielsson, 1999, p. 517). Comparisons of timing as a measurement of difficulty are therefore problematic, especially if comparisons are made in different tempos.

Instead of timing deviations, I decided to take tempo limits as a measurement of performance difficulty in the study: a polyrhythmic configuration playable in a smaller range of tempos could be considered more difficult or complex. In order to draw plausible conclusions regarding which factors influenced performance difficulty, a comprehensive set of exercises was needed. I therefore designed the exercises in the study to include several aspects which might influence performance difficulty, based on the assumption that various metrical levels in the rhythmic configurations interact in shaping the structure and playable tempo range of the rhythm.

Despite the study's focus on defining polyrhythmic complexity experientially rather than theoretically, complexity in the mathematical sense is a useful term for comparing rhythmic configurations. In the thesis, a rhythmic level with more played strokes is considered more complex as long as the rhythm is not congruent with the pulse. However, since there is no straight-forward way of weighting the influence of different number of pulses, complexity can in this sense only be used to compare rhythms played over the same number of pulses.

1.5.4 Limitations

This thesis is generally situated in the Western classical tradition with a focus on solo percussion performance. I focus on performance as a practical skill involving processes distinct from concert performance (Schick, 1994, pp. 132–134). Therefore, my practical study does not focus on interpretative or aesthetic considerations connected with specific pieces of music. Although as an exploratory study, all conclusions are preliminary, this study aims to provide a foundation for future studies and practical applications.

My personal limitations in playing affect the results of this study and limit its generalisability. In percussion performance, high tempo limits have been shown to vary between individuals; Mclaughlin & Boals (2010) tested the fastest playable tempos for 27 percussion students and found that although the effect of the mental organisation was consistent across participants, the attained tempos were tied to the individual. Although my attained tempos in the exercises might reflect limitations in my own playing, the results can be used to find tendencies across exercises, which can act as starting points for future research.

Similarly, because the preferred strategies and interpretations of polyrhythmic configurations can vary between individuals (Grieshaber, 1990, pp. 143–148; Handel, 1984, p. 481), my preferred strategies for practicing polyrhythms may not be universally efficient. In addition, the strategies used in this study to cope with the polyrhythms are intended for a practice situation; in a concert situation or in ensemble playing, effective strategies may differ. Furthermore, these practice strategies for polyrhythms only apply to percussion instruments; since musical understanding is connected to the instrument played, the same strategy may not work for every instrument.

In addition, a self-observational study holds a risk of unreliability (Williamon et al., 2021, p. 89). It is difficult to draw a strictly defined standard for "good enough" performance, which complicates the matter of choosing a non-arbitrary cut-off for sub-standard playing. To counteract this risk, I took notes during each practice session, writing down my experiences in the moment to avoid relying on recollection. In addition, I made video recordings of a sample of exercises in order to check for consistent quality of performance.[9] With these methods, I ensured that the results of the self-observational study were as consistent as possible.

[9] These video recordings are in personal possession and can be obtained upon request. Contact information is included in Appendix 3.

2. Polyrhythmic Types and Pedagogical Approaches

In this section, musical examples of polyrhythms are examined and categorized according to their rhythmic qualities and structures. [10] For this purpose, the musical examples are examined through the lens of the metrical hierarchy and grouping structure theory in *A Generative Theory of Tonal Music* (Lerdahl & Jackendoff, 1983). The implications of these examples regarding possible tempo limits are also considered. Approaches in pedagogical material are then described and shown to be appropriate for the practice of certain categories of polyrhythms. Performance strategies used in the practical study are also derived from the pedagogical material. In the discussion of polyrhythmic types (section 2.2), the hierarchical time-span polyrhythm, together with elements from Magadini's (1993) pedagogical approach, are found to be best suited for the purposes of my practical study and therefore have informed the design of the polyrhythmic exercises used in the practical section of the thesis.

2.1 Polyrhythmic Types

2.1.1 Extrametrical Polyrhythms

The music of Chopin often features polyrhythmic elements, with many examples to be found in his Nocturnes. These polyrhythms can be rather lengthy, spanning whole measures, as in Figure 5, or short, as in Figure 6. The polyrhythms in Chopin's music often have the character of a melodic embellishment or a virtuosic ornamental flourish and are meant to be played freely over the meter rather than being strictly rhythmical, with considerable room for rubato (David Bruce Composer, 2019; Lerdahl & Jackendoff, 1983, p. 72).[11] Due to the ornamental character, I call this type of polyrhythm an *extrametrical polyrhythm.*

[10] The examples I have played or practiced myself are Chopin's *Nocturne Op. 9, No.1*, Sandström's *Drums*, Cage's *Third Construction*, Takemitsu's *Rain Tree*, Zappa's *The Black Page*, and Stout's *Two Mexican Dances No. 2*.

[11] The 5:3 polyrhythm in Figure 6 is perhaps a borderline case, which could be either extrametrical or played with stricter rhythm depending on interpretation. At any rate, the ornamental character of the melodic figure should be taken into account.

Figure 5

Chopin (1832/ca. 1905), Nocturne Op. 9, No. 1*, measures 4–6.*

Figure 6

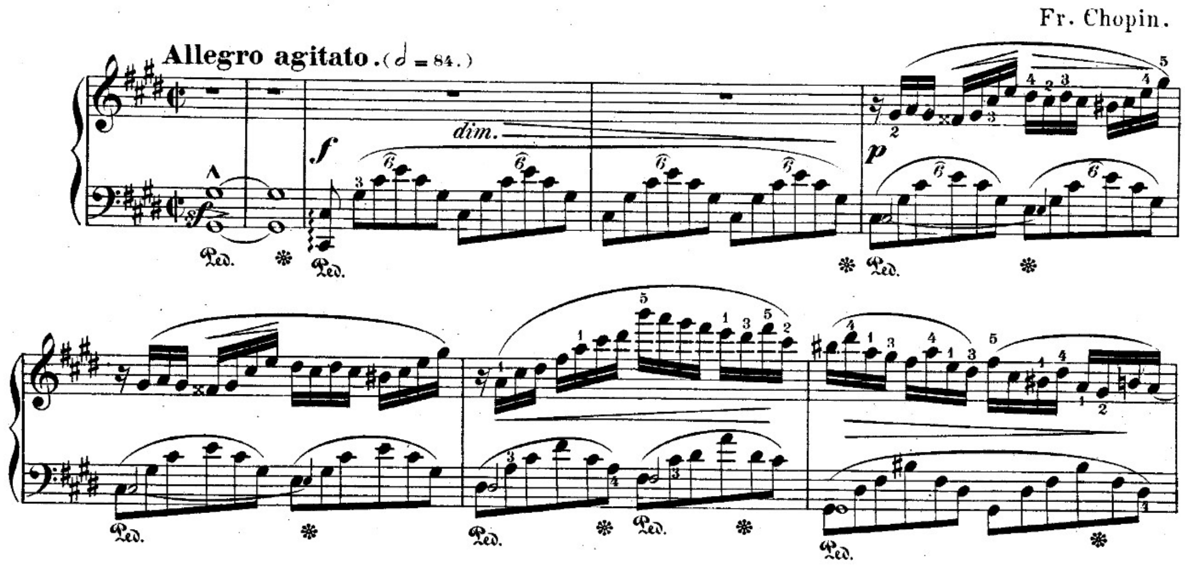
Chopin (1832/ca. 1905), Nocturne Op. 9, No. 3*, measures 9–11.*

2.1.2 Beat-Division Polyrhythms

When polyrhythms are used more consistently, as is the case in Chopin's *Fantaisie-Impromptu* (Figure 7), the polyrhythm has a more genuinely rhythmic character, creating a greater need for precise playing. According to David Bruce (2019), the purpose of the polyrhythm in *Fantaisie-Impromptu* is to "to create a sort of manic energy" (5:09); it therefore fulfils a different musical purpose than the previous examples. In this music, the polyrhythm is created by the interaction of the sixteenth note melody and the triplet accompaniment. The pulse is felt on the half-note level, but if played slowly for practice, it might also be felt on the quarter-note level. In either case, the melody and accompaniment coincide on every beat. Therefore, I call this type of polyrhythm a *beat-division polyrhythm*.

Figure 7

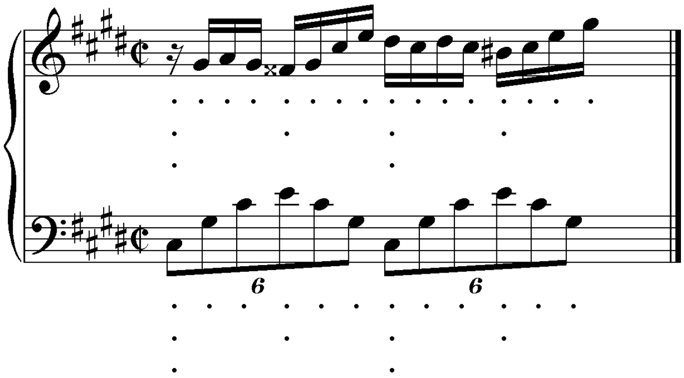
Chopin (1855/1880), Fantaisie-Impromptu, Op. 66*, measures 1–8.*



Beat-division polyrhythms, by definition, have a shared pulse level but two different subdivisions. Each voice forms a normal subdivision in relation to the experienced pulse; the polyrhythm is, in Lerdahl & Jackendoff's (1983) terminology, created by the interaction of two incongruent metrical structures at subtactus levels (in Fantaisie-Impromptu, the melody and accompaniment [Figure 8]). As the subdivisions divide a single beat, the pulse tempo must be within a comfortable range so that the pulse is experienced as *tactus*, i.e. the primary metrical level (Lerdahl & Jackendoff, 1983, p. 71). The beat-division polyrhythms are therefore limited to a relatively short time-span.

Figure 8

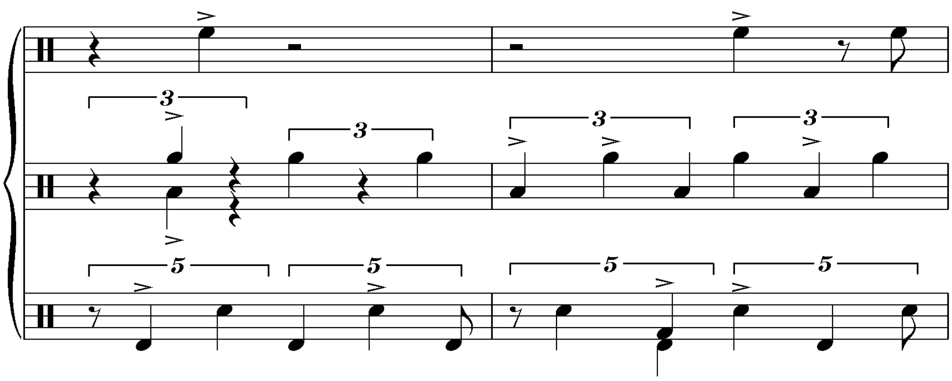
Chopin (1855/1880), Fantaisie-Impromptu, Op. 66*, measure 5, with metrical grid notation.*



An extreme example of beat-division polyrhythms is found in Xenakis' *Persephassa* for six percussionists: the performers are supposed to play two or even three divisions at the same time by themselves (Figure 9). In other sections of the piece, there are simultaneous subdivisions of two, three, five, six and nine notes per beat in various combinations and figurations, divided between the members of the ensemble.

Figure 9

Xenakis (1970), Persephassa*, Percussion F, measures 96-97.*

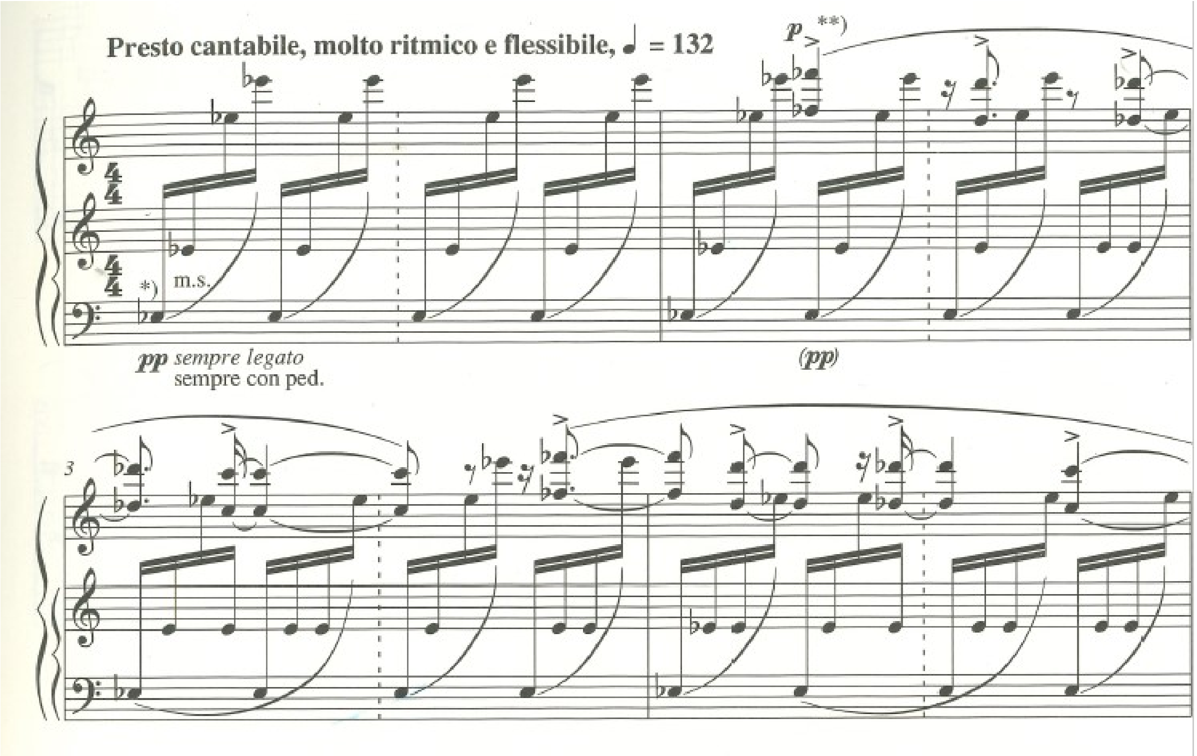


2.1.3 Phrase-Length Polyrhythms

Another approach for creating polyrhythms is found in Ligeti's sixth etude, "Automne à Varsovie" (Figure 10). Here, a melody with a five-note duration is superimposed on a repeating four-note figure. Ligeti's technique for creating polyrhythms does not divide the pulse but is created from a fast common subdivision, which ties the polyrhythm together (David Bruce Composer, 2019). As these polyrhythms create phrases that can stretch out over long spans of time, I call them *phrase-length polyrhythms*.

Figure 10

Ligeti (1986), "Automne à Varsovie*," measures 1–4.*



Another example of phrase-length polyrhythms is found in Sven-David Sandström's *Drums*. Towards the end of the piece, each performer plays irregular two- and three-note groupings, which eventually form increasingly stable patterns. At rehearsal letter R, the first part has a seven-note grouping pattern, both part two and three have a five-note grouping pattern, part four has an eleven-note grouping pattern, and the timpani part has a thirteen-note grouping pattern (Figure 11).

Figure 11

Sandström (1984), Drums*, measures 370–371, with added grouping structure and metrical grid notation.*

Each part forms a polyrhythmic grouping pattern against the quarter-note pulse, but considering the clear sixteenth-note subdivision and the groups of two or three notes, the rhythms can also be considered as syncopated patterns.[12] Polyrhythmic patterns are also formed in the relations between the parts.[13] For example, the relation between parts one and two can be described as a 7:5 pattern, tied together by the common subdivision, but in such a dense texture these relations are scarcely heard, and the 7:5 ratio is rather a description of the music as written.

This type of polyrhythm is sometimes called polymeter. The distinction made between polyrhythm and polymeter is that polyrhythms consist of two or more incongruent subdivisions within a shared time-span (i.e. beat-division polyrhythms), whereas polymeters

[12] If there is a meaningful difference, it is perhaps one of stability: irregular groups are simply syncopated, whereas repeating patterns can be considered polyrhythmic.
[13] Note that the relation between part two and three is not polyrhythmic, but only a reversal of the two-plus-three grouping.

consist of incongruent time-spans connected by a shared pulse or subdivision (i.e. phrases of different length) (Brubaker et al., 2009, p. 19; Signals Music Studio, 2018).[14] However, meter is defined as a mode of attending – a mental construct not written in the music. As we necessarily attend to rhythm through a single metric framework, there is in fact no such thing as a polymeter (London, 2012, p. 67; Poudrier & Repp, 2013). Moreover, in *Drums*, the polyrhythmic patterns are made by the note groupings, whereas the meter is common for all parts.[15] For these reasons, it makes more sense to describe passages such as the one in *Drums* as a type of polyrhythm.

Since the groups can be made in any size larger than the smallest duration, the technique of using a common subdivision as a reference can be used as a compositional tool to create polyrhythmic cycles of practically infinite length. Messiaen uses this technique in the first movement of his *Quartet for the End of Time*, where the cello and piano part both have a repeating "rhythmic pedal" (Messiaen's terminology) and the piano part contains a repeated melody that does not match this rhythmic pedal (Griffiths, 1985, p. 91). Since the rhythmic cycles of the cello and piano parts are of unequal length, the cycles would have to repeat for approximately 230 minutes before coinciding again (Pople, 1998, p. 26).

Similarly, so-called long-ranging or structural polyrhythms have been used extensively by Elliot Carter as a form-generating compositional tool, constructing entire movements on this principle. In the song *O Breath*, Carter combines three different subdivisions and note groupings to create a very complex structural polyrhythm – equivalent to quintuplets in groups of 37 notes, sextuplets in groups of 43 notes, and nonuplets in groups of 65 notes[16] (Coulembier, 2012, p. 20). This technique can be seen as a combination of the beat-division and the phrase-length techniques, in which different subdivisions create implied polyrhythms over the pulse, and groupings of the subdivisions create polyrhythms over longer time-spans.

[14] Signals Music Studio (2018) explicitly uses the wording "phrases of different length" to describe polymeters, which of course corresponds to the wording "phrase length polyrhythms".

[15] Agawu (2003, pp. 79–85) pursues similar arguments for rejecting polymeter as a useful concept in the study of African music.

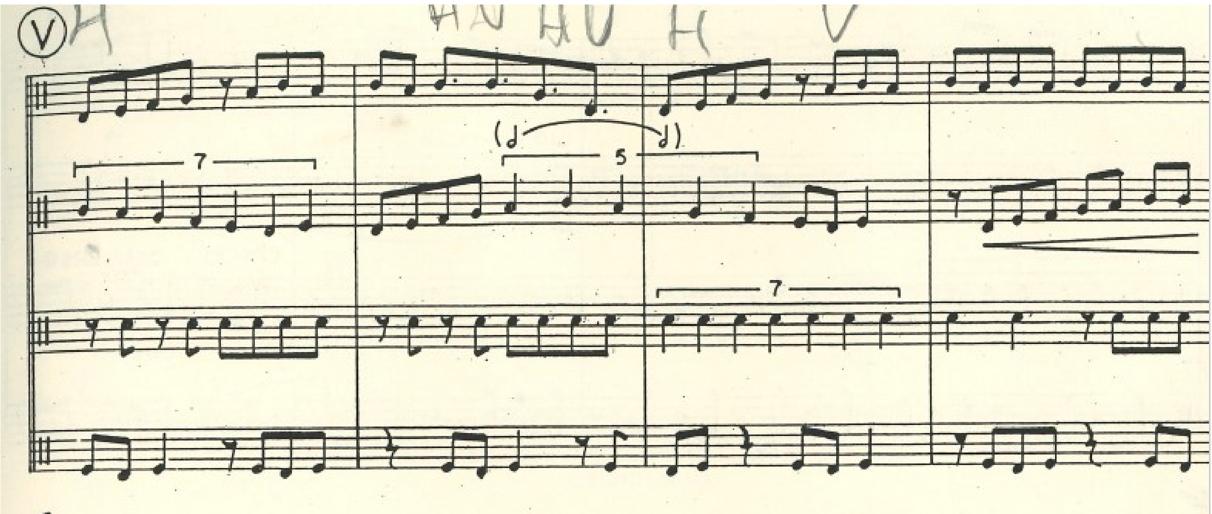
[16] The music is notated in 12/8, and the rhythms are notated as sixteenth notes, dotted sixteen-note quintuplets, and sixteenth-note triplets in the score.

2.1.4 Time-Span Division Polyrhythms

In John Cage's *Third Construction* for four percussionists, yet another type of polyrhythm is found (Figure 12). The septuplet and the quintuplet rhythms both span a full measure, but the quintuplet rhythm is displaced to the middle of the measure. Since the pulse in this piece is felt on the half-note level, the rhythms neither divide the beat nor are tied together by a common subdivision. Because the polyrhythms in this example divide two beats, they can be considered an extension of the beat-division polyrhythm from the beat level to the measure level. However, the dotted eighth notes in the second measure of the example form a 4:3 polyrhythm over the span of three quarter notes. With the pulse as half notes, it divides neither the measure, the pulse, nor any multiple of the pulse. Because this type of polyrhythm is not directly connected to an experienced pulse but can in principle divide any time-span, I call it a *time-span division polyrhythm*.

Figure 12

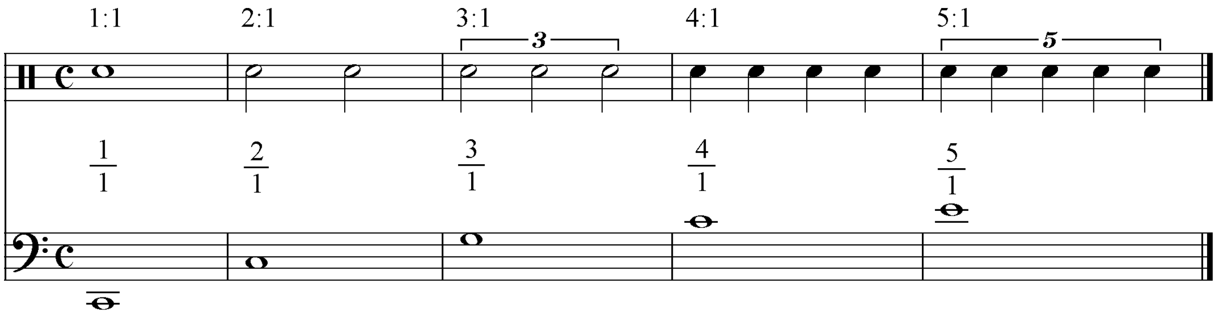
Cage (1970), Third Construction*, p. 45.*



The time-span division polyrhythm was likely first theorised by Henry Cowell (1930/1996) in his book *New Musical Resources*, which has been influential for many subsequent composers, such as Conlon Nancarrow (Gann, 1995, pp. 1–2). Using the overtone series as a conceptual foundation, the whole note is taken as a fundamental note value, which is divided into smaller note values, similar to the division of the fundamental tone into partials in the overtone series (Figure 13).[17]

Figure 13

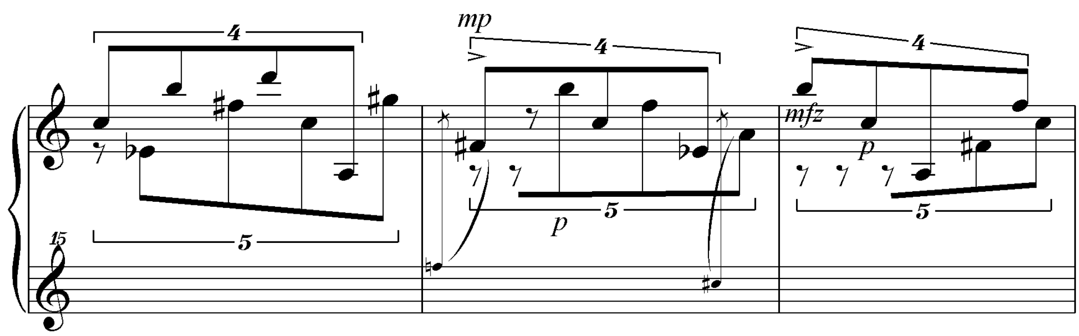
Rhythmic ratios and corresponding partials in the overtone series.



In the example from Cage's *Third Construction*, each performer plays only a single line, in which the polyrhythms are equally spaced over the duration of the rhythm. In other pieces, there are more varied time-span division polyrhythms. In Takemitsu's *Rain Tree,* for vibraphone and two marimbas, the vibraphone solo consists of a repeating 5:4 pattern broken into irregular figures (Figure 14), creating a free and unmeasured feeling in the music.

Figure 14

Takemitsu (1981), Rain Tree*, Percussion C (vibraphone), p. 4.*



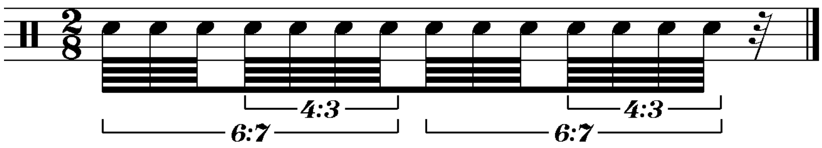
The possibilities offered by time-span division polyrhythms can even exceed human cognitive capacities. For example, Ferneyhough's *Bone Alphabet*, written for percussionist Steven Schick, contains several highly intricate rhythmic passages. One such passage is

[17] The notation of the ratios in the overtone series is borrowed from Nicholson & Sabat (2018).

replicated in Figure 15, in which several layers of polyrhythms are nested within a single rhythmic figure.[18] In Schick's (1994, p. 138) article about learning the piece, he writes, "How does one simultaneously think six in the time of seven and four in the time of three? The answer occurred to me in a flash: one does not!"

Figure 15

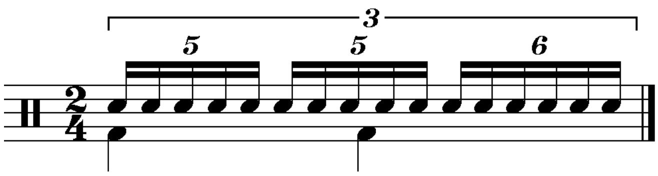
Nested rhythm found in measure 2 of Ferneyhough, Bone Alphabet.



The technique of nesting rhythms, however, can be used in more playable ways.[19] One example is found in Frank Zappa's *The Black Page* (Figure 16).[20] Here, quintuplets and sextuplets are fitted within a quarter-note triplet. Compared to the nested rhythms in Ferneyhough, these rhythms have a clearer connection to the pulse, taking up the duration of two quarter notes.

Figure 16

Nested rhythm found in Zappa, The Black Page *(transcribed).*



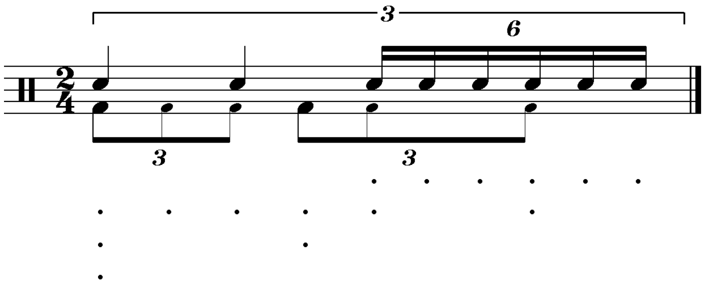
The rhythm in Figure 16 has a clear hierarchical organisation, but the interaction of the rhythmic layers is difficult to pin down theoretically. The framing 3:2 rhythm and the embedded sextuplets can reasonably well be fitted into a metric grid by adding a triplet subdivision (Figure 17). The quintuplets, however, are more problematic. Since they neither fit the triplet subdivision nor align with the pulse, they cannot be described in metrical grid

[18] In the actual music, there is also another line of 32nd-note triplets above the 6:7 line, adding yet another layer of difficulty.
[19] See Shawn Crowder (2020) for a good example of nested rhythms on the edge of playability.
[20] I do not have access to any readily available printed version.

notation without violating Lerdahl and Jackendoff's (1983, p. 72) "metrical well-formedness rules": "Every beat at a given level must also be a beat at all smaller levels present at that point in that piece." To be sure, the metrical grid is not designed for explaining polyrhythms, but it can nevertheless be easily adapted to explain both beat-division and phrase-length polyrhythms. Theoretically problematic metrical structures are generally a feature of time-span division polyrhythms.

Figure 17

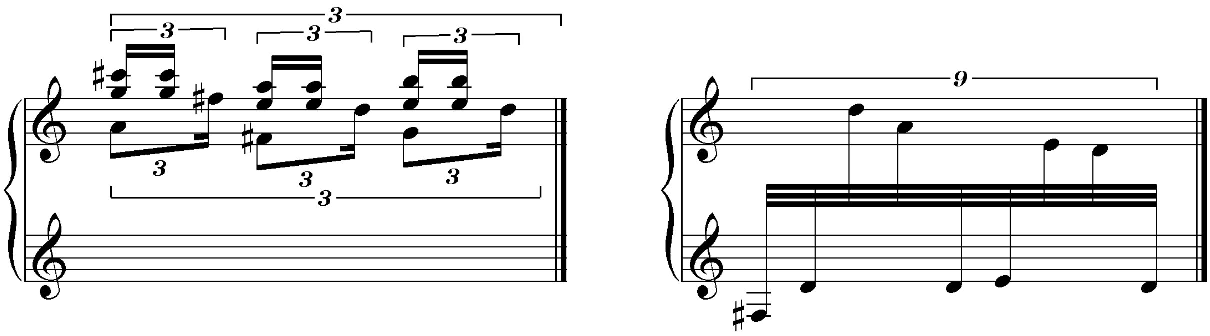
Nested 3:2 rhythm with metrical grid notation.



The two 9:2 polyrhythms in Stout's *Two Mexican Dances for Marimba* (previously mentioned in section 1.2.1) both divide two beats and can be categorised as time-span division polyrhythms. Yet the two rhythms have different grouping structures and expressive qualities. The nested rhythm in the first measure of Figure 18*,* with its clear and even hierarchical structure, is similar to the rhythm from Zappa's *The Black Page*. The rhythm in the second measure, with a 4+5 grouping structure, lacks an even division and therefore has a less clear hierarchy.

Figure 18

Two polyrhythms from Stout's (1977) Two Mexican Dances No. 2, *measures 43 and 66.*



For this reason, a distinction can be made between *hierarchical polyrhythms* and *non-hierarchical polyrhythms*. Hierarchical polyrhythms are organised in a basic polyrhythm, the *framing rhythm*, which is elaborated by an *embedded subdivision*. Since the hierarchical organisation requires an even division on two levels (i.e. the framing rhythm and the embedded subdivision), the configurations that can form hierarchical polyrhythms are restricted to combinations of lower prime numbers.[21] Non-hierarchical polyrhythms, on the other hand, are structured in unevenly divided note groups. As the non-hierarchical structure is freer than the hierarchical structures, polyrhythms that cannot be evenly divided are conceivable, such as 13:3 (see Figure 1).

2.2 Discussion of Polyrhythmic Types

As previously mentioned, beat-division polyrhythms require that an experienced beat is divided. In principle one could postulate a two-second limit as the longest duration in which this is possible, but in such slow tempos, one voice is most likely heard as the pulse rather than a subdivision, and the actual limit is likely significantly shorter. For this reason, there is a limited set of subdivisions which can be used to create beat-division polyrhythms – perhaps not more than seven or eight notes per beat, given that the subdivisions must be within a playable tempo range while at the same time be felt as a single group.[22]

[21] Compare section 3.2. For present purposes, I do not consider the possibility of irregular rhythmic figures as an elaboration of the framing rhythm.
[22] In other words, an 8:6 rhythm is not effectively felt as two groups of 4:3 rhythms.

The phrase-length polyrhythm is versatile. It can be used to create syncopated patterns and overlapping groupings, as well as long-ranging polyrhythms far beyond immediate comprehension. It is, however, limited to a playable common subdivision; more complex configurations tend to be more conceptual than genuinely rhythmic. As phrase-length polyrhythms are based on a common subdivision, they generally become more difficult in higher tempos. For this reason, they can be considered an opposite to beat-division polyrhythms, which, as they are based on divisions of the pulse, become more difficult in slow tempos.

The time-span division polyrhythms can be considered to lie conceptually somewhere in between the beat-division and phrase-length types: a time-span of a certain *length* (other than the pulse) is *divided* to form the polyrhythm. This feature lends the time-span division polyrhythms great possibilities for nuanced rhythmic structures but also runs a risk of excessive complexity. As this type of polyrhythm has a direct connection neither to the pulse nor to a common subdivision, it is difficult to describe the metrical framework of time-span division polyrhythms, and it is unclear how tempo affects the playability of this type of polyrhythm.

Time-span division polyrhythms can be created in several ways. One approach, exemplified in the music of Ferneyhough (see Figure 15), takes a note value as a starting point and can be called a note-value-based approach. While this approach can create mostly any polyrhythmic configuration, it can for the same reason be overly complicated and difficult to structure. Although this method can be used,[23] it is difficult to systematise. Therefore, the method is more appropriate for learning specific rhythms than for a systematic study.

The other approach is to divide time-spans of multiple pulses. This approach has the advantage of being grounded in an experienced pulse rather than a written note value. Within this approach, the non-hierarchical variant features unequal grouping structures over the time-span, whereas the hierarchical variant features a basic polyrhythm used as a framing rhythm for embedded subdivisions.

The unequal groupings in non-hierarchical polyrhythms can be used to create variations and nuance in the structure of a single polyrhythmic configuration. This is

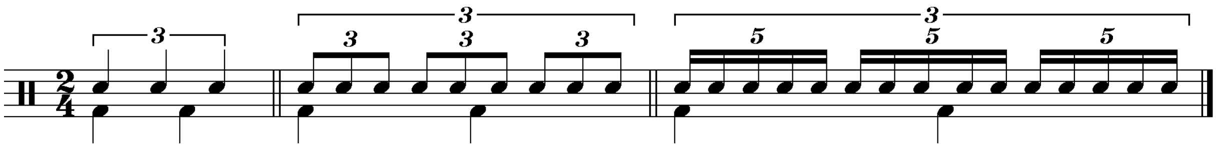
[23] Shawn Crowder (2020) shows how this concept can be used in a practice situation.

reflected in Holmquist's (2010) method (see section 2.3.2), which contains full pages of variations for each polyrhythmic configuration. However, the many ways of structuring the polyrhythm makes each rhythm stand on its own; because there is no recursiveness in the structure, they are, like the note-value-based approach, better suited for learning specific musical passages than systematic study.

In a hierarchical polyrhythm, many configurations can be based on a single structure. For instance, the 3:2 polyrhythm can be used as a framing rhythm for the 9:2 and 15:2 polyrhythms (Figure 19). Hierarchical polyrhythms are therefore recursive and have a clear connection to the pulse. The hierarchical organisation defines clear levels in the rhythmic configuration, allowing an examination of how the interaction of the metrical levels influence performance for various configurations in different tempos.

Figure 19

The 3:2 framing rhythm with embedded subdivisions.



2.3 Pedagogical Approaches

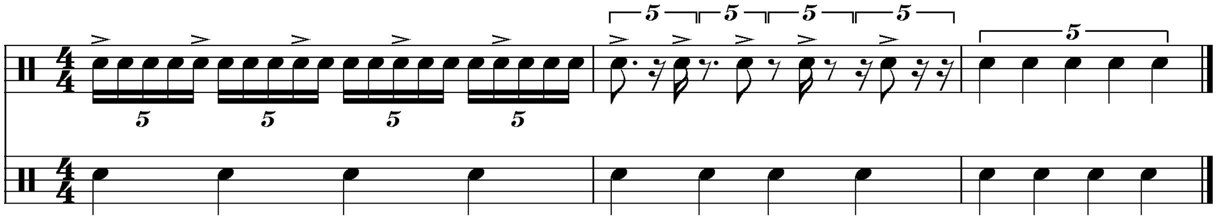
Having identified and discussed four different types of polyrhythms (extrametrical polyrhythms, beat-division polyrhythms, phrase-length polyrhythms, and time-span division polyrhythms), this section now turns toward identifying four pedagogical approaches towards practicing and performing polyrhythms.

2.3.1 The Subdivision Approach

A common method for explaining polyrhythms is the subdivision approach, which connects polyrhythmic lines by a subdivisional grid. This approach goes by different names in the literature, such as the least common denominator strategy (Bortz, 2003; Schick, 1994) or partial elimination (Chaffee, 1976, p. 55). The approach is exemplified in Figure 19, showing the steps for learning the 5:4 polyrhythm. First, the common subdivision is played and the notes forming the final polyrhythm are accented (measure 1). If the goal is to play the composite pattern, the grid can be removed after some practice (measure 2), leaving only the accented notes that form the basic polyrhythm (measure 3).

Figure 20

Steps for learning the 5:4 polyrhythm according to the subdivision approach.

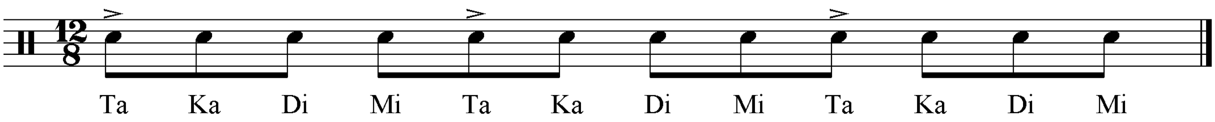


In some pedagogical material, the subdivision approach is used as an explanation or in preparatory exercises (Chaffee, 1976; Jersild, 1975; Magadini, 1993; Palmqvist, 2006), whereas others make more extensive use of the approach (Rissman, 2005; Wood, 2013). Since the subdivision approach is based on a common subdivision, it is most closely associated with phrase-length polyrhythms, although it can also be used for practicing simple time-span division polyrhythms (Chaffee, 1976; Magadini, 1993; Palmqvist, 2006). However, if the target tempo is too fast for the common subdivision to be felt, the subdivision strategy is not very effective; Chaffee stresses that the goal in practice is to avoid reliance on the subdivision, as it cannot be used in fast tempos and tends to make one's playing rigid and dull (Chaffee, 1976, p. 55). This tempo limitation restricts the use of the subdivision strategy for more complex polyrhythms, where it is often more effective as a visual aid than as a practice method (Bortz, 2003; Schick, 1994).

A related method to the subdivision strategy is to use syllables, such as the takadimi system, in such a way that the syllable pattern does not match the pulse (Figure 21) (Wood, 2013). By using a set pattern of syllables for all note groups of equal length (i.e. ta-ka-di-mi always correspond to a four-note group), it is possible to superimpose mostly any note group over a given subdivision. The method is well-suited for phrase-length polyrhythms, particularly for those in which the subdivision is explicitly played.

Figure 21

The 4:3 rhythm in polyrhythmic syllables, adapted from Wood (2013, p. 66).



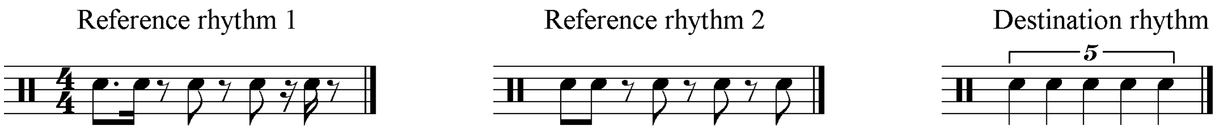
Another variant is to use a catchphrase: a simple sentence or a few words that match the composite pattern of the polyrhythm, such as "pass the golden butter" or "where did you work today" for the 4:3 polyrhythm. The benefit of this approach is that the stresses of the words naturally focuses on one part of the rhythm – "pass the golden butter" stresses the threes and "where did you work today" stresses the fours (Grieshaber, 1990, p. 143; Pianist Magazine, 2016). Shifting between the phrases then gives a good understanding for both sides of the rhythm. The method is, however, limited to simple polyrhythms and works best in a moderate tempo.

2.3.2 The Approximation Approach

Another approach is what I call the approximation approach, primarily used in *The Reference Rhythm Method* by Mulle Holmquist (2010). The first step in this approach is to learn a rhythmic pattern, the "reference rhythm", which consists of the same number of notes and beats as the "destination rhythm", i.e. the polyrhythm to be learned. In Holmquist, the reference rhythm is based on an eighth-note or a sixteenth-note subdivision. When the reference rhythm is mastered, one tries to smoothen it out over the duration of the rhythm until it can be evenly played. The reference rhythm should resemble the destination rhythm but can take various forms. To give an example, the two reference rhythms for the 5:4 polyrhythm are presented in Figure 22.

Figure 22

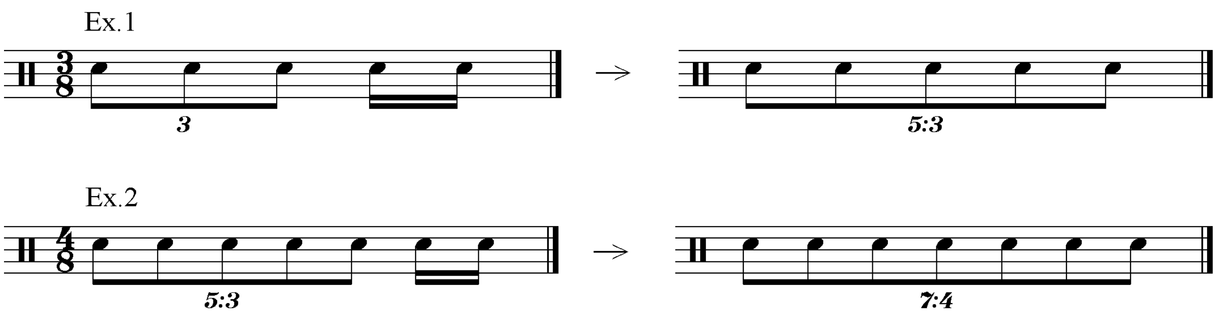
The 5:4 reference rhythms and destination rhythm, adapted from Holmquist (2010, p. 29).



A variant of the approximation approach is found in Jersild (1975). As in Holmquist, a rhythmic pattern resembling the intended polyrhythm is first learned and then smoothened out. Instead of basing the reference rhythm on a single subdivision, Jersild proposes using a combination of triplets and sixteenth notes as an approximation for the 5:3 polyrhythm (Figure 23, Ex. 1). He continues in the same manner by using a combination of the 5:3 polyrhythm followed by two sixteenth notes to approximate the 7:4 polyrhythm (Figure 23, Ex. 2).

Figure 23

The approximation approach for the 5:3 and 7:3 polyrhythms in Jersild (1975).



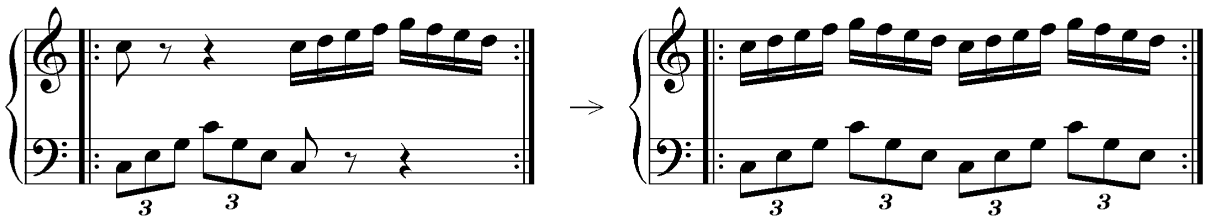
Since the reference rhythm in Holmquist is usually based on a sixteenth-note subdivision, the approximation approach works best in a medium range tempo, in which the sixteenth notes can be comfortably played. The choice of reference rhythm lends a certain feeling or phrasing to the final polyrhythm. Holmquist (2010, p. 5) notes that one can choose to play with the “jagged” reference rhythm, the “smooth” polyrhythm, or any timing variation in between. Thus, there is no neutral pattern in the approximation approach, and care must be taken to choose an appropriate reference rhythm if the method is to be used for learning a specific passage in a piece. As the groupings of the reference rhythms can be varied, the approximation method is perhaps best suited for non-hierarchical time-span division polyrhythms.

2.3.3 The Shift-and-Merge Approach

What I call the shift-and merge approach is mainly used by pianists, as it focuses on two-handed playing in fast tempos. In this approach, one hand plays the slow part of the polyrhythm, and the other hand plays the fast. First, one should shift back and forth between the hands, using simple and well-known patters, such as scales or arpeggios (Figure 24, first measure), before trying to merge the rhythms together by playing both hands at the same time (Figure 24, second measure) (Informance, 2019). The shift-and-merge approach is well-adapted for learning fast and short polyrhythms, such as beat-division polyrhythms. While it is a specifically pianistic approach it could perhaps be adapted to collaborative polyrhythms by splitting the voices between two performers.

Figure 24

Steps for learning the 4:3 polyrhythm according to the shift-and-merge approach.



2.3.4 The Framing Rhythm Approach

What I term the framing rhythm approach is most thoroughly implemented in Magadini's *Polyrhythms: The Musician's Guide* (1993).[24] The strategy entails learning a basic polyrhythm, such as 3:2, 3:4, or 5:4, and then using the basic polyrhythm are as a framing rhythm for embedded subdivisions. Figure 25 shows an example of the principle, where eighth notes and eighth-note triplets are embedded within a framing 3:2 and 5:2 polyrhythm.

[24] It is, however, mentioned in Palmquist (2006, p. 162) and Jersild (1975, p. 21) regarding the 9:2 polyrhythm and disregarded as "too mathematical" in Holmquist (2010, p. 3).

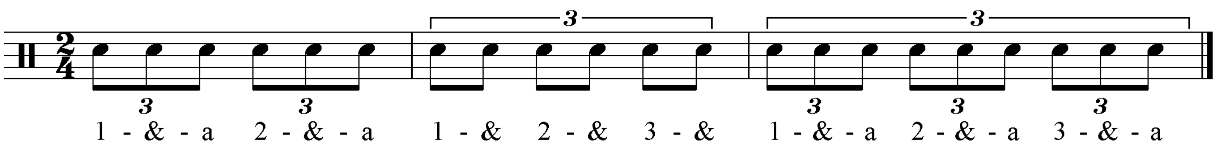
Figure 25

Framing rhythms with embedded subdivisions.

The framing rhythm is presented through a common subdivision (Figure 26, measure 1), but Magadini stresses that one should count according to the intended polyrhythm and not the subdivision (Figure 26, measure 2). Therefore, the basic polyrhythm is counted in the same way as the pulse is normally counted, in some way imposing another metrical layer over the pulse. More elaborate embedded subdivisions are then counted the same way as they would be in a common metrical context (Figure 26, measure 3). Magadini only treats polyrhythms over four-beat measures and embedded subdivisions up to sixteenth notes, but in principle the strategy is applicable to any playable meter, framing rhythm and embedded subdivision, making it particularly well suited for hierarchical time-span division polyrhythms.

Figure 26

Example of the counting system in Magadini (1993).



2.4 Performance Strategies in the Pedagogical Approaches

The following section contains a short discussion of performance strategies derived from the pedagogical approaches. I define performance strategies as mental organisations of rhythms that aim to facilitate performance. This categorization and discussion of performance strategies contributes to the theoretical basis of the study, as appropriate tempo ranges of these strategies were examined in the practice sessions of this study.

As a performance strategy, the subdivision approach ties together polyrhythmic configurations through a mental subdivision. While mental subdivisions are not well suited for high tempos, they could be stabilising in slow tempos. I refer to this strategy as the *subdivision strategy*.

The approximation approach can be effective for phrasing the polyrhythm, but the focus on reshaping a rhythm to the intended polyrhythm is specifically a learning tool and does not present any particular performance strategy.

As a performance strategy, the shift-and-merge approach uses a common slow beat as a timing goal to stabilise the polyrhythm.[25] Even though the approach is focused on two-handed playing, the strategy of mentally organising the polyrhythm to be felt over a single beat can be used in single-line polyrhythmic practice by mentally organising metronomic pulses into a higher-level tactus. Since the strategy relies on grouping pulses into a single tactus, it should work best in high tempos. I refer to this strategy as the *beat-division strategy*.

In the framing rhythm approach, the polyrhythm is guided by both the underlying pulse and the played rhythm, in some way adding another metrical layer over the pulse. In this strategy, it is not clear whether performance is influenced more by the length of the whole pattern or the duration between the elements. The interrelated control of the timing makes predictions on a suitable tempo range difficult. Since this strategy relies on perceiving the rhythm as a whole by integrating all layers into a single composite pattern, I refer to this strategy as the *composite-pattern strategy.*

In addition, *motion-based strategies* were also used in the practice sessions. These were not derived from pedagogical material but were informed by the close connection between rhythm and motion (Bengtsson, 1987, p. 71), as well as the fact that the motions involved in playing change with tempo (see section 1.2.2). Several motion-based strategies were used, including a focus on hand motion, a focus on the downbeat (either every beat or the cycle), continuity of motion, and ways of creating a stable sense of pulse through motion.

[25] The term “common slow beat” is adapted from Kvifte (2007, p. 75), there referred to as “common slow pulse”.

2.5 Application to the Practical Study

From the polyrhythm types and pedagogical approaches discussed above, I have chosen hierarchical time-span division polyrhythms and Magadini's (1993) framing rhythm approach as the basis for the construction of the exercises in the practical study. Hierarchical time-span division polyrhythms, together with the framing rhythm approach, not only provide a clear method for practicing increasingly complex polyrhythmic configurations but also presents a way of organising the material in such a way that more complex configurations are based on simpler configurations. In addition, the study combines the concept of polyrhythm types with the concept of performance strategies by identifying effective performance strategies for hierarchical time-span division polyrhythms at different tempos.

3. Practical Study

The practical study aims to explore how polyrhythmic configurations relate to tempo limits and how tempo influences the polyrhythmic structure. For the study, I wrote a large set of exercises based on hierarchical time-span division polyrhythms and informed by the theoretical concepts discussed in Chapter 2; I then practiced these exercises in a wide range of tempos, documenting the resulting data. This chapter describes the design of the exercises, including a description of their hierarchical features and related measurements of tempo; the equipment used in the practice sessions; methods of data collection; methods of conducting the practice sessions; and methods of result analysis.

3.1 Hierarchy in Time-Span Division Polyrhythms and Tempo Measurements

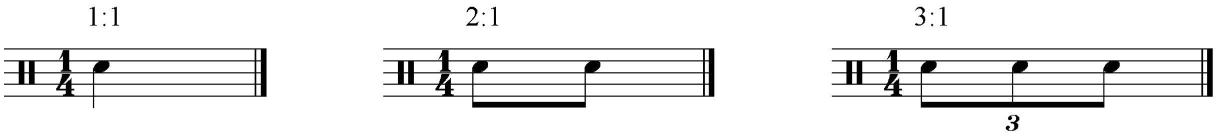
A sense of tempo is not determined by a single metrical level but is a sum effect of the interactions between various rhythmic levels (London, 2012, p. 33). In designing the exercises, I therefore assumed that any level in the hierarchy of the rhythmic configuration and their interactions might influence the tempo limits and the structure of the polyrhythms. For this reason, the exercises were designed to include many aspects which might influence performance. The following section aims to provide a clear description of terms used in the practical study that relate to hierarchical aspects of time-span division polyrhythms and tempo measurements.

3.1.1 Ratios

In the exercises, the note value corresponding to the pulse is always written as a quarter note in music notation. For clarity within the text, however, the rhythms are referred to as ratios, written as *X:Y* (X against Y), where X represents the number of played notes in the rhythm and Y represents the number of unplayed pulses in the rhythm. A 1:1 ratio is notated in quarter notes, a 2:1 ratio in eighth notes, a 3:1 ratio in triplets, and so forth (Figure 27).

Figure 27

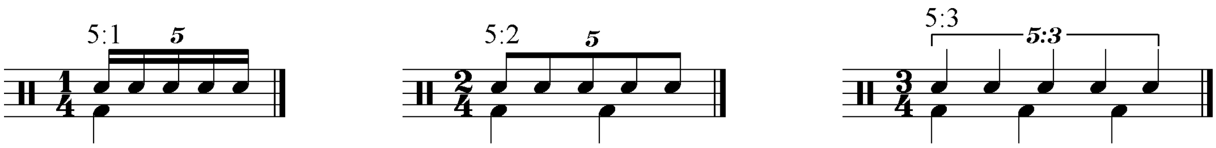
Note values and their corresponding ratios.



The number of pulses delimits the relative time-span of the rhythm, called the *cycle*. A rhythm spanning one beat has a one-beat cycle, a rhythm spanning two beats has a two-beat cycle, and so on. In X:Y form, a two-beat cycle is written as X:2. Figure 28 shows rhythms with five played notes within a one-beat cycle, a two-beat cycle, and a three-beat cycle, together with the corresponding ratios. The upper voice indicates the played rhythm, and the lower voice indicates the pulse.

Figure 28

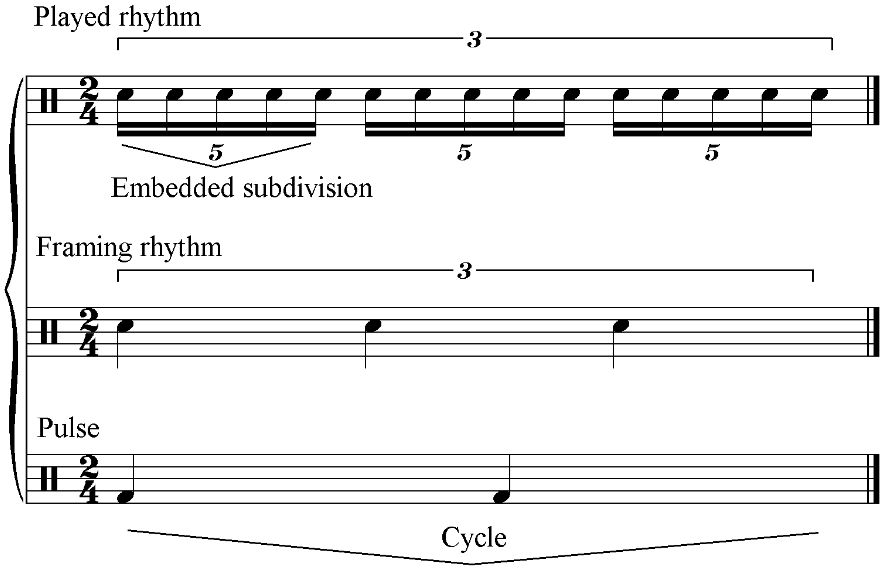
Quintuplet rhythms over pulse cycles of various lengths.



The played notes in the polyrhythms are organised in a grouping structure by the *framing rhythm* and the *embedded subdivision*. Although various grouping structures are often possible, they are always evenly divided across the cycle. In Figure 29, the *played rhythm* has fifteen notes, divided into three groups of five notes. In rhythms with embedded subdivisions, X stands for the framing rhythm. Since the rhythm spans two beats, the framing rhythm is written as 3:2. The embedded subdivisions are indicated by superscript numbers in connection with X, resulting in a general form of X^n:Y. The rhythm in Figure 29 is therefore written as 3^5:2.

Figure 29

Hierarchical levels in the 3^5:2 polyrhythm.



3.1.2 Length and Density

The number of played notes is referred to as the *length* of the rhythm. A rhythm with more played notes is *longer,* and a rhythm with fewer played notes is *shorter*. The length of the rhythm is not affected by the number of pulses. Therefore, all the quintuplet rhythms in Figure 28 have the same length. In polyrhythms with embedded subdivisions, the length of the rhythm corresponds to the multiple of the framing rhythm and the embedded subdivision (X*n). X corresponds to the length of the rhythm when embedded subdivisions are not considered. Therefore, the rhythm in Figure 29 can also be described as a 15:2 rhythm.

In this study, density refers to the number of notes per beat.[26] Density thus describes the speed of the rhythm relative to the pulse. For the same pulse tempo, a higher density corresponds to a faster rhythm. Therefore, rhythms with a high density are only playable in correspondingly lower tempos. While the two quintuplet rhythms in Figure 30 have the same length, the density of the 5:2 rhythm is only half the density of the 5:1 rhythm.

[26] Density is also used in certain literature to describe the number of notes per second (Dowling, 1986, p. 185).

Figure 30

Comparison of density in two quintuplet rhythms.

3.1.3 Categorisation

The X:Y form can be used to categorise groups of rhythms with similar characteristics. For instance, all rhythms with a two-beat pulse cycle can be grouped under the X:2 category. The exercises with a two-beat cycle are thus referred to as the *X:2 set* (of exercises). The same principle can be applied to any aspect of the configuration. For instance, 4:Y indicates all rhythms with four played notes (over any number of pulses). Rhythms with embedded subdivisions are indicated by X^n:Y. Therefore, X:Y stands for rhythms without embedded subdivision.

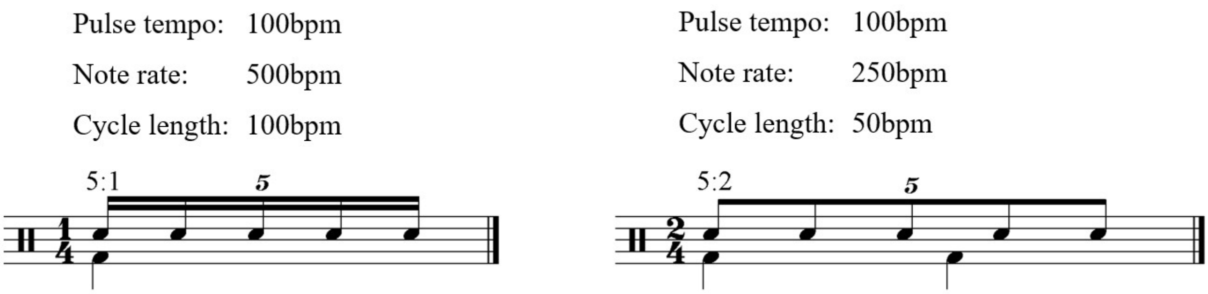
3.1.4 Tempo Measurements

As any level of the hierarchy can influence performance, several different types of tempo measurements are used. Primary measurements of tempo in this study are *pulse tempo*, which refers to the metronome's tempo setting; *note rate*, which refers to the tempo of the played notes; and *cycle length*, which refers to the duration required for a rhythmic cycle to repeat. The pulse tempo is the primary tempo measurement to which the others relate: note rates can be calculated by multiplying the density of the rhythm with the pulse tempo, and cycle length can be calculated by dividing the pulse tempo with the number of pulses. Figure 31 shows an example of the primary tempo measurements for the 5:1 and 5:2 rhythms in a 100bpm pulse tempo.[27]

[27] A table showing the relation between the tempo measurements in further detail is included in Appendix 1, Table B.

Figure 31

Tempo relations in the 5:1 and 5:2 rhythms.



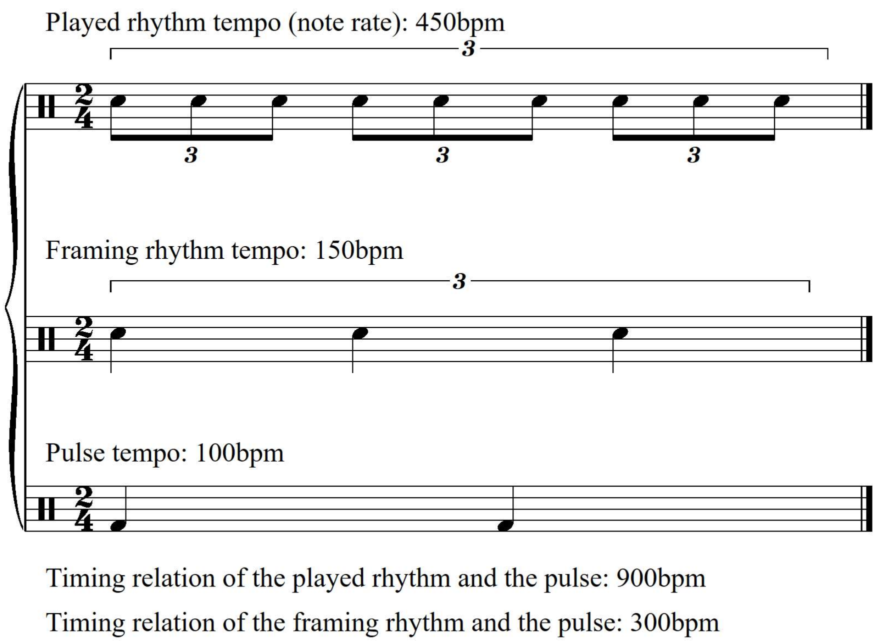
Secondary tempo measurements include the tempo of the framing rhythm, the timing relation of the played rhythm and the pulse, the timing relation of the framing rhythm and the pulse, and the shift rate between rhythms.

The *timing relation* corresponds to the note rate of the implied subdivision[28] (i.e. the least common multiple of the two rhythms in a polyrhythm) of either the played rhythm and the pulse, or the framing rhythm and the pulse. The tempo of the framing rhythm can be calculated by multiplying the density of the framing rhythm with the pulse tempo. The timing relation between the played rhythm and the pulse can be calculated by multiplying the length of the rhythm with the pulse tempo. The timing relation between the framing rhythm and the pulse can be calculated by multiplying the length of the framing rhythm with the pulse tempo. The two timing relation measurements (played rhythm and pulse, framing rhythm and pulse), together with the tempo of the framing rhythm, are exemplified for the 3^3:2 rhythm in 100bpm pulse tempo in Figure 32.

[28] The implied subdivision is much the same as the common subdivision (see section 2.3.1). The distinction I make is that the common subdivision is experienced, while the implied subdivision refers to the abstract relation between two parts of the subdivision.

Figure 32

Secondary tempo measurements in the 3^3:2 polyrhythm.



Since each exercise consists of two coupled rhythms, the measurement of *shift rate* also applies. The shift rate refers to the duration between the starting notes of two different rhythms; the shift rate is therefore affected by how many times the rhythms are repeated. The shift rate can be calculated by dividing the cycle length with the number of repetitions. More repeats before shifting to the other rhythm corresponds to a slower shift rate, as shown in Figure 33.

Figure 33

Shift rates in the 1:1-2:1 exercise.

3.2 Design

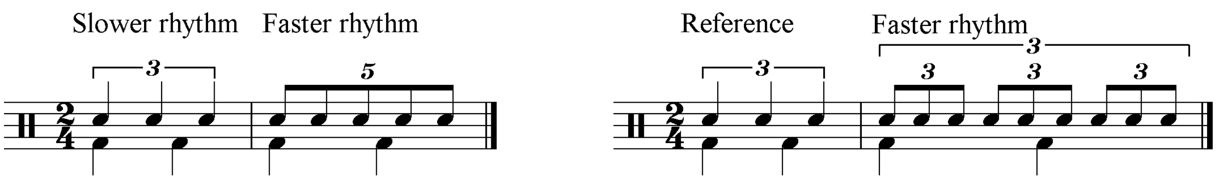
This section discusses the characteristics of the exercises used in the practical study, describing in detail the characteristics of all sets of exercises. In total, there are 100 exercises arranged in six different sets – the X:1 set, the X:2 set, the X^n:2 set, the X:3 set, the X^n:3 set, and the X:4 set – according to the number of pulses in the cycle and the inclusion of embedded subdivisions.

All exercises were designed to be played as single-line polyrhythms. In pedagogical material, most pedagogical approaches (such as the subdivision approach, the approximation approach, and the framing rhythm approach) can be adapted for both single-line and two-handed playing. Therefore, a study of single-line polyrhythms is closer to what music pedagogy has seen as important in polyrhythmic playing and offers a wider definition of polyrhythms than commonly found in academic literature. The design was mostly informed by Magadini's (1993) framing rhythm approach, but elements from other pedagogical materials also influenced the design.

The exercises are composed of two rhythms per exercise. While most studies on polyrhythm have tested a single polyrhythm for each trial, music does not generally consist of ever-repeating pulse trains. The juxtaposition of two rhythms results in a more musical exercise and also shows the influence of the combination of rhythms on both tempo limits and structure. In the exercise, the first rhythm is always the slowest. In exercises without embedded subdivisions, it is called the *slower rhythm*; in exercises with embedded subdivisions, it is called the *reference*.[29] In both types of exercises, the second rhythm is called the *faster rhythm*. Figure 34 provides an illustration of this terminology.

Figure 34

Labels of the two rhythms in the exercises.



In music, the played rhythm is generally more varied than the pulse, which often remains constant throughout the entire piece. Therefore, within each exercise the number of pulses (i.e. the pulse cycle) is kept constant, while the rhythm played over the measure changes. The exercises can be categorized as containing one-, two-, three-, or four-beat

[29] Not to be confused with reference rhythms in Holmquist (2010).

pulse cycles. The exercises over one-beat cycles do not form polyrhythmic configurations but were used as a control group against which to compare the results of the other exercises.

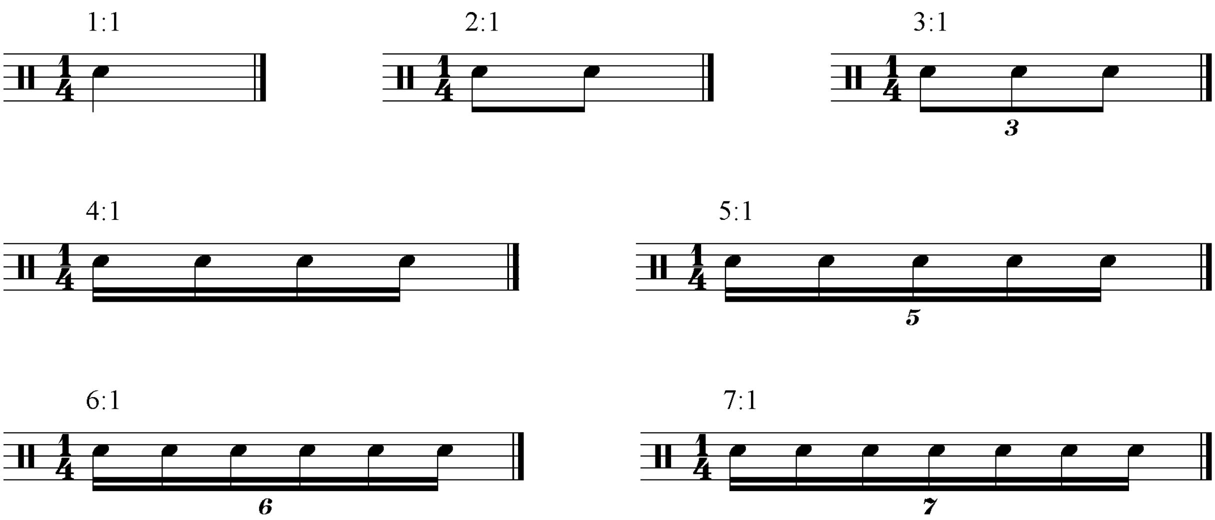
While the pulse cycle was limited to a maximum of four beats (X:4), the framing rhythms were limited to a maximum division of septuplets (7:Y) for the sake of limiting the number of exercises in the study.[30] Exercises with embedded subdivisions were similarly limited to embedded septuplets (X^7:Y). In addition, rhythms with a density higher than ten strokes per beat were excluded due to the rhythmic instability of larger prime numbers.

3.2.1 The X:1 Set

The first set of exercises consists of rhythms based on subdivisions ranging from the pulse to septuplets, played over a pulse cycle of one beat. Figure 35 shows the rhythms notated together with their corresponding ratio.

Figure 35

X:1 rhythms used to create the X:1 set of exercises.

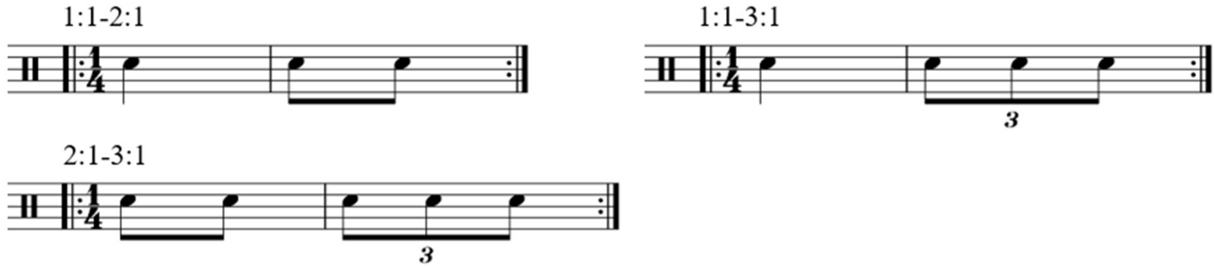


To make exercises, the rhythms were combined in every possible order from slower to faster, so that every possible shift between the rhythms was tested.[31] Figure 36 shows how the 1:1, 2:1, and 3:1 rhythms can be coupled in this manner to form three two-bar exercises.

[30] The septuplet limit follows suggestions from Jersild (1975, p. 29).
[31] A similar approach is found in Chaffee (1976), where all subdivisions are combined into preliminary exercises. The focus on shifting between rhythms is also a vital part of the shift-and-merge approach, discussed in section 2.3.3.

Figure 36

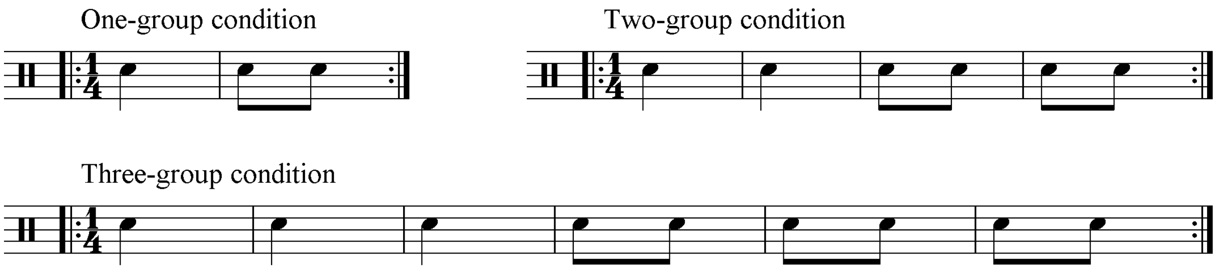
Combination of X:1 rhythms into X:1 exercises.



In the exercises in Figure 36, each rhythm is played once before shifting to the other. I call this arrangement the *one-group condition*. Several exercises with the one-group condition are referred to as *one-group exercises*. In addition to the one-group condition, the study also tested *two-* and *three-group conditions*, in which each rhythm was played twice or three times before shifting to the other. Figure 37 shows the 1:1–2:1 exercise with one-group, two-group, and three-group conditions. Different group conditions were used to test the effect of shift rates on the playable tempo range.

Figure 37

The 1:1–2:1 exercise with various group conditions.



The two- and three-group conditions of some exercises corresponded exactly to the one-group conditions of other exercises; some exercises were therefore omitted. For instance, the two-group 1:1–3:1 exercise was effectively equivalent to the one-group 2:1–6:1 exercise, prompting the omission of the 2:1–6:1 exercise from the study (Figure 38). However, to check if this assumption of equivalency was correct, the 2:1–4:1 exercise (in which the one-group condition was equivalent to the two-group condition of the 1:1-2:1

exercise) was included in the study. The remaining exercises included in the study are shown in Table 1.

Figure 38

Equivalency between two X:1 exercises.

Table 1

All exercises in the X:1 set.

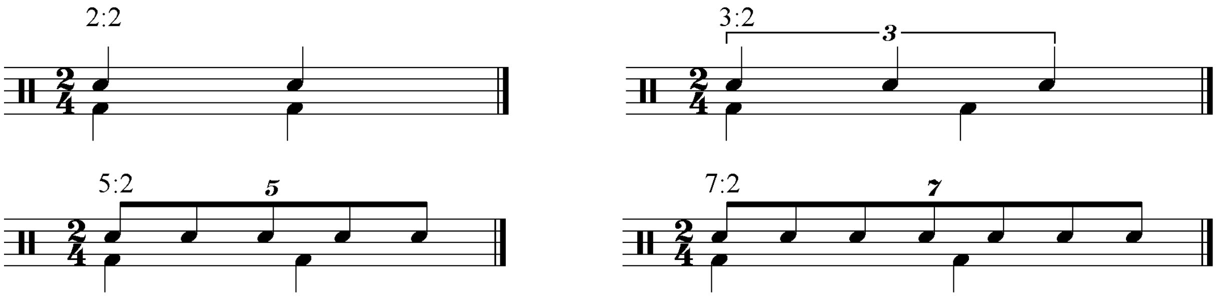
	2:1	3:1	4:1	5:1	6:1	7:1
1:1	1:1–2:1	1:1–3:1	1:1–4:1	1:1–5:1	1:1–6:1	1:1–7:1
2:1		2:1–3:1	2:1–4:1	2:2–5:1	excluded	2:1–7:1
3:1			3:1–4:1	3:1–5:1	excluded	3:1–7:1
4:1				4:1–5:1	excluded	4:1–7:1
5:1					5:1–6:1	5:1–7:1
6:1						6:1–7:1

3.2.2 The X:2 Set

The next set of exercises consists of polyrhythms with a two-beat pulse cycle. The 2:2 rhythm, equalling the pulse, was also included. The set thus included four rhythms, shown in Figure 39. These rhythms were combined into six exercises, shown in Table 2.

Figure 39

X:2 rhythms used to create the X:2 set of exercises.



Note. The upper voice represents the played rhythm, while the lower voice represents the pulse.

Table 2

All exercises in the X:2 set.

	3:2	**5:2**	**7:2**
2:2	2:2–3:2	2:2–5:2	2:2–7:2
3:2		3:2–5:2	5:2–7:2
5:2			5:2–7:2

3.2.3 The X^n:2 Set

In the next set of exercises, the X^n:2 set, the 3:2 and 5:2 rhythms were used as framing rhythms for embedded subdivisions. Figure 40 shows the 3:2 framing rhythm with possible embedded subdivisions and their ratios.

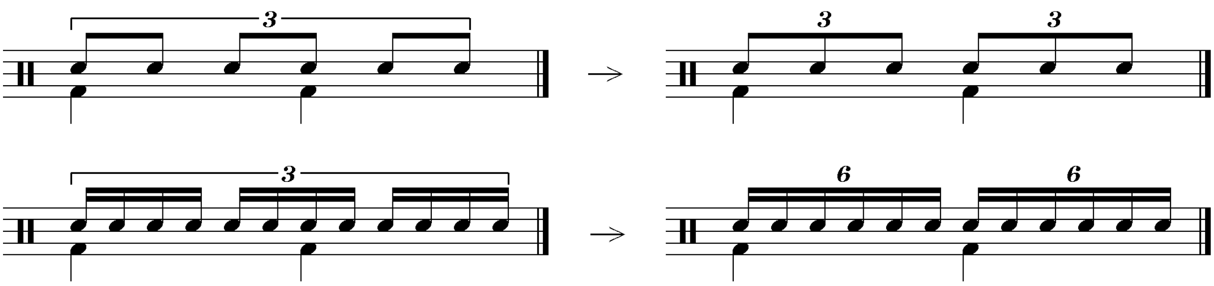
Figure 40

The 3:2 framing rhythm with various embedded subdivisions.

In the cases of 3^2:2 and 3^4:2, the rhythms correspond to normal subdivisions; Jersild (1975, p. 21) describes these configurations as "accent polyrhythms".[32] However, these rhythms, shown in Figure 41, were excluded due to their close relation to the pulse. 7^n:2 rhythms were also excluded due to their high density and limited practical applicability.

Figure 41

Accent polyrhythms (3^2:2 and 3^4:2) excluded from the study.



With these exclusions in place, the complete set of X^n:2 rhythms includes three rhythms: 3^3:2, 3^5:2, and 5^3:2 (Figure 42). Regarding 3^5:2 and 5^3:2, both rhythms consist of fifteen strokes, and the only difference between them is the grouping structure. By practicing both groupings, it was possible to examine whether the grouping structure influenced the playability of an otherwise similar rhythm.

[32] In Danish, *betoningspolyrytmer*.

Figure 42

X^n:2 rhythms used to create the X^n:2 exercises.

To create X^n:2 exercises, the X^n:2 rhythms were combined with the four X:2 rhythms (2:2, 3:2, 5:2, 7:2). Had the rhythms with embedded subdivisions been combined with each other, it would result in too many exercises.[33] Since the X:Y rhythms form the framing rhythms in the X^n:Y configurations, the X:2 rhythms were taken to represent the X^n:2 rhythms. The X:2 rhythms thus acted as the *references* to the X^n:2 rhythms. The advantage of this arrangement was that each rhythm was given four references that were consistent across the entire set. The exercises could therefore be examined and compared to see how different references affected the playability and stability for each rhythm. This approach produced four exercises for each X^n:2 rhythm, twelve in total, shown in Table 3.

Table 3

All exercises in the X^n:2 set.

	3^3:2	**3^5:2**	**5^3:2**
2:2	2:2–3^3:2	2:2–3^5:2	2:2–5^3:2
3:2	3:2–3^3:2	3:2–3^5:2	3:2–5^3:2
5:2	5:2–3^3:2	5:2–3^5:2	5:2–5^3:2
7:2	7:2–3^3:2	7:2–3^5:2	7:2–5^3:2

Besides being categorised by their rhythmic configuration (i.e. 2:2, 3:2, 5:2, 7:2), the references were also categorised by their function in the exercise. The reference always belonged to one of four categories:

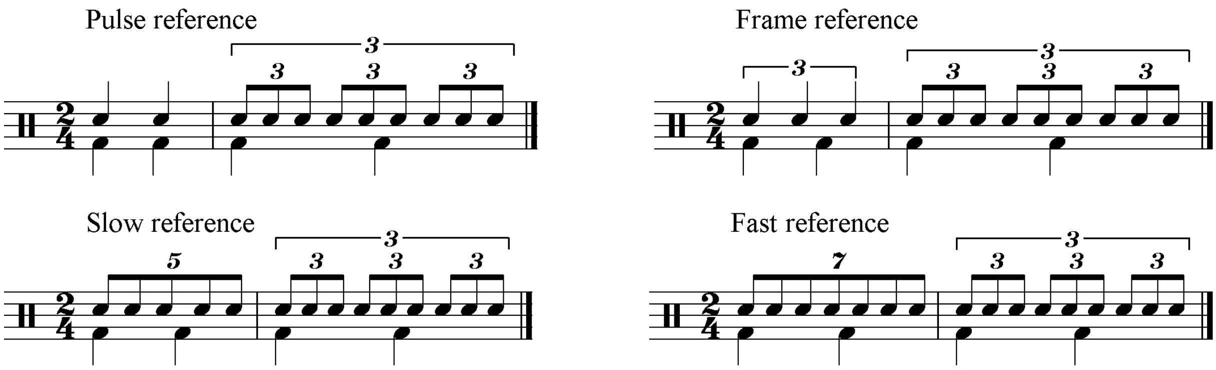
[33] For the X^n:2 set, this was not a pressing issue, but the X^n:3 set would include far too many exercises. To make the conditions of the X^n:Y sets similar, both sets included the use of X:Y rhythms as references.

1. *Pulse reference* (i.e. references equal to the pulse)
2. *Frame reference* (i.e. references congruent with the framing rhythm)
3. *Slow reference* (i.e. references not congruent with pulse or framing rhythm and slow relative to the fast reference)
4. *Fast reference* (i.e. reference not congruent with pulse or framing rhythm and fast relative to the slow reference)

The category to which a reference belongs depends in part on the configuration of the other rhythm in the exercise, i.e. whether the reference rhythm matches the framing rhythm or not. The references for the 3^3:2 exercises and their appropriate categories are shown in Figure 43.

Figure 43

Types of references for the 3^3:2 exercises.

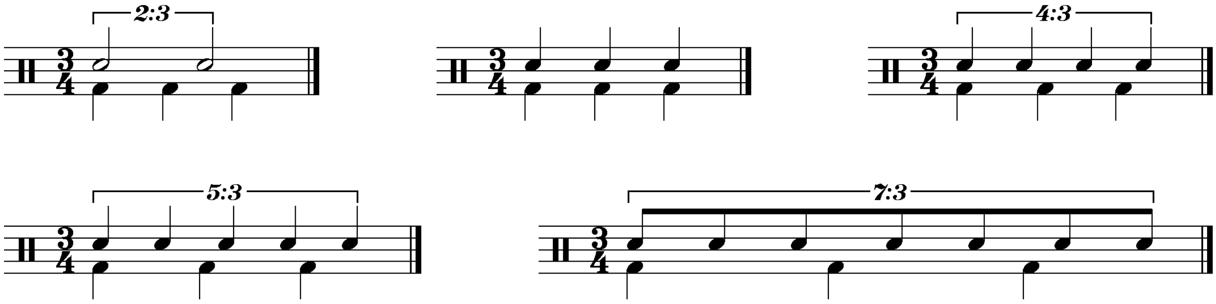


3.2.4 The X:3 Set

The X:3 exercises were constructed similarly to the X:2 exercises. The rhythms 2:3, 3:3, 4:3, 5:3 and 7:3 (Figure 44) were combined into two-bar exercises, ten in total, shown in Table 4. The 4:3 rhythm is included despite being a duple subdivision of 2:3 (and thus could be considered a 2^2:3 rhythm), as it is experienced rather differently than 2:3 and is common enough to be regarded as a basic polyrhythm in its own right.

Figure 44

X:3 rhythms used to create the X:3 exercises.



Note. The notation in Figure 44 is somewhat irregular. In Sibelius music notation software, the 2:3 rhythm would normally be notated as quarter notes. Similarly, 7:3 would normally be written as 7:6 eighth notes. In the context of this thesis, the logic of notation is tied to a sounding or experienced pulse. Therefore, notating 7:3 according to common practice as 7:6 would imply a different rhythm. For this reason, the notation has been altered to change in accordance with duple subdivisions of the pulse. Rhythms equal or faster than the pulse are written as quarter notes; 2:1 rhythms and faster are written as eighth notes; and so forth.

Table 4

All exercises in the X:3 set.

	3:3	**4:3**	**5:3**	**7:3**
2:3	2:3–3:3	2:3–4:3	2:3–5:3	2:3–7:3
3:3		3:3–4:3	3:3–5:3	3:3–7:3
4:3			4:3–5:3	4:3–7:3
5:3				5:3–7:7

3.2.5 The X^n:3 Set

Because the X^n:3 set was by far the largest of all sets in the study, the rhythms are here presented only in Table 3.5 and not in notation. Some possible rhythms were omitted: the 2^2:3 rhythm was excluded as it was equal to the 4:3 rhythm, and the 5^7:3, 7^5:3, and 7^7:3 rhythms were excluded as their density was greater than ten strokes per pulse.

Table 5

X^n:3 rhythms used to create the X^n:3 exercises.

n=	2	4	5	7
2:3	excluded	2^4:3	2^5:3	2^7:3
4:3	4^2:3	4^4:3	4^5:3	4^7:3
5:3	5^2:3	5^4:3	5^5:3	excluded
7:3	7^2:3	7^4:3	excluded	excluded

To create X^n:3 exercises, the X^n:3 rhythms (Table 5) were combined with the X:3 rhythms (2:3, 3:3, 4:3, 5:3, 7:3), as in the X^n:2 set. However, the inclusion of both the 2:3 and 4:3 rhythms as X:3 rhythms caused a problem: if both were included as references, each X^n:3 rhythm would have five references, making a total of sixty exercises – an unrealistically high number. After some testing, the 4:3 rhythm was chosen as a reference for most of the exercises. However, the 2:3 rhythm was the reference for the 2^n:3 exercises, since it was congruent with the framing rhythm. The complete set of X^n:3 exercises is shown in Table 6.[34]

Table 6

All exercises in the X^n:3 set.

	$2^4/4^2$:3	**2^5:3**	**2^7:3**	**4^4:3**	**4^5:3**	**4^7:3**
2:3	2:3–$2^4/4^2$:3	2:3–2^5:3	2:3–2^7:3			
3:3	3:3–$2^4/4^2$:3	3:3–2^5:3	3:3–2^7:3	3:3–4^4:3	3:3–4^5:3	3:3–4^7:3
4:3				4:3–4^4:3	4:3–4^5:3	4:3–4^7:3
5:3	5:3–$2^4/4^2$:3	5:3–2^5:3	5:3–2^7:3	5:3–4^4:3	5:3–4^5:3	5:3–4^7:3
7:3	7:3–$2^4/4^2$:3	7:3–2^5:3	7:3–2^7:3	7:3–4^4:3	7:3–4^5:3	7:3–4^7:3

	5^2:3	**5^4:3**	**5^5:3**	**7^2:3**	**7^4:3**
3:3	3:3–5^2:3	3:3–5^4:3	3:3–5^5:3	3:3–7^2:3	3:3–7^4:3
4:3	4:3–5^2:3	4:3–5^4:3	4:3–5^5:3	4:3–7^2:3	4:3–7^4:3
5:3	5:3–5^2:3	5:3–5^4:3	5:3–5^5:3	5:3–7^2:3	5:3–7^4:3
7:3	7:3–5^2:3	7:3–5^4:3	7:3–5^5:3	7:3–7^2:3	7:3–7^4:3

[34] Due to their almost equivalent grouping structure, the 2^4:3 and 4^2:3 exercises were tested together and are therefore written as a single exercise (see section 4.5.2).

3.2.6 The X:4 Set

The last set consists of X:4 rhythms. The 4:4, 3:4, 5:4 and 7:4 rhythms, shown in figure 3.11, were combined into six two-bar exercises. These six exercises are shown in table 3.7. Although it would have been preferable to also include X^n:4 exercises, time limitations prohibited me from testing these exercises.

Figure 45

X:4 rhythms used to create the X:4 exercises.

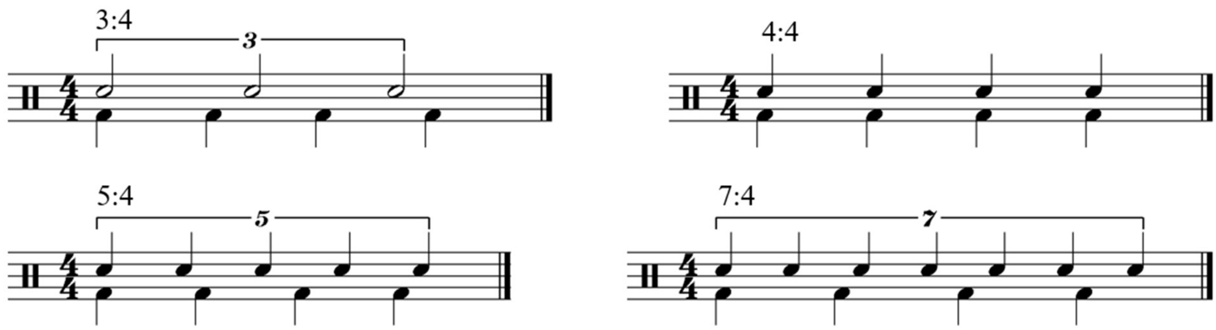


Table 7

All exercises in the X:4 set.

	4:4	5:4	7:4
3:4	3:4–4:4	3:4–5:4	3:4–7:4
4:4		4:4–5:4	4:4–7:4
5:4			5:4–7:4

3.3 Equipment

The exercises were practiced on a snare drum or a Wincent Slimpad practice pad with Vic Firth SD2 Bolero sticks. (For practical reasons, I could not always use the same snare drum; see section 1.5.4.) A Korg Beatlab metronome (BTL-1) was used with headphones (Vic Firth headphones, Urbanears Plattan headphones, and Beats headphones depending on location). Video recordings were made with a Samsung Galaxy A21s smartphone.

3.4 Data Collection

Data was primarily gathered through written notes during each practice session. Note categories included the following:

- Starting tempos
- Highest and lowest attained tempos
- Stability of performance in specific tempos
- Performance strategies

Performance strategies included feeling a pulse-level tactus (in polyrhythmic exercises corresponding to a composite-pattern strategy), grouping pulses into a multiple-pulse tactus (in polyrhythmic exercises corresponding to a beat-division strategy), subdivision strategies, and motion-based strategies.

Notes which did not relate to any of the mentioned categories but arose in the moment were also taken. These notes included the following: experience of fatigue or tiredness, good flow, comfortable tempo ranges (i.e. tempos in which the exercises were easy to play), clear timing relations between the played rhythm or the framing rhythm and the pulse, timing difficulties (untight or rubato), cognitive difficulties (blurred notes or pulses), and the added pulse strategy.

When exercises were replayed to test the reliability of the results, video recordings were made of these exercises to double-check my timing in high tempos. The videos are in personal possession and can be obtained upon request.[35]

3.5 Procedure

3.5.1 Organisation of the Practice Sessions

The exercises were practiced in the order of the design as presented in section 3.2. A metronome was used at all times during the practice sessions, and the exercises were repeated many times without stopping in each tempo. The practice sessions generally lasted around twenty to thirty minutes, with at most three sessions in one day; since tempo ranges varied between exercises, some sessions required more time than others. In cases when I experienced too much fatigue or tiredness, the sessions were ended and restarted the following day.

[35] Contact information is included in Appendix 3.

3.5.2 Tempo Increments and Decrements

Each exercise was started in a comfortable tempo and sped up or slowed down to extreme tempos. The starting tempo varied for each exercise, depending on the density of the faster rhythm and whether high tempos (fast tempo exercises) or low tempos (slow tempo exercises) were aimed for. Sessions with fast tempo exercises were started in lower tempos, and sessions with slow tempo exercises were started in higher tempos, so that the full tempo range was tested.

For most exercises, tempo was increased or decreased in increments of 10bpm. For exercises with relatively fast rhythms, increments of 5bpm were used so that the increase in note rate would correspond more closely to the increase of the relatively slower rhythms.[36] For the X:2 and X:4 exercises, increments of both 10bpm and 20bpm were used, as the longer cycles made the note rate increase less for the same changes of tempo. For the X:3 exercises, increments of 15bpm were used.

In low tempos, the tempo was decreased according to set conditions. The tempo was lowered in decrements of 10bpm down to 60bpm, thereafter in increments of 5 bpm. In tempos below 40bpm, the tested tempos were 37, 35, 32, 30, 27.5, 25, 22.5, 20, and 17.5bpm.

In the fast tempo exercises, all group conditions were practiced in each tempo before the tempo was increased. In the slow tempo exercises, there was no apparent difference between one-group, two-group, and three-group exercises, so all slow tempo exercises were played only in the one-group condition.

3.5.3 Issues Related to the Metronome

In high tempos, the metronome's downbeat articulation was turned off, so that only the pulse was heard. In tempos above 300bpm (the highest tempo setting on the metronome), the metronome articulated eighth notes at a half-speed tempo setting; for example, a setting of 150bpm would sound at 300bpm. For the X:3 set, the metronome articulated triplets with the tempo setting at a third of the sounding pulse. In these cases when the downbeat articulation was turned off, I often played with my eyes closed to

[36] For example, a tempo increment of 10bpm corresponds to an increased note rate of 20bpm for the 2:1 rhythm and an increased note rate of 70bpm for the 7:1 rhythm.

ensure that I did not accidentally cheat by following the metronome's visual representation of the downbeats of each cycle (a red blinking light). Since it was difficult to ascertain that I played the exercise accurately in all repetitions, video recordings were used to double-check that I always played in time with the metronome.

For tempos below 30bpm, I turned off the sound of the pulse articulation so that the metronome articulated only the downbeats in a two-beat period, which effectively cut the tempo in half. In these cases, I closed my eyes to avoid cheating by following the green pulse light.

3.5.4 Compromises

According to the original plan of the study, all exercises were to be practiced in both tempo extremes. However, due to increasing fatigue in my left arm (see section 1.5.2), I could not practice all exercises in fast tempos. As a compromise, I tested only the X:1 and X:2 set and some exercises in the X:3 and X:4 in both high and low tempos, and the exercises with embedded subdivisions were only tested in slow tempos. Exercises that were only played in slow tempos were started with a pulse tempo between 140–160bpm; in this tempo range, pulses could be grouped into a multiple-pulse tactus, thus allowing for testing of the tempo range in which this strategy could be used. Exercises which could not be played in these tempos (i.e. exercises with a higher density than approximately four notes per beat) were instead started with a note rate between 525–607bpm.

3.6 Data Analysis

3.6.1 Data Storage

The data from the written notes were stored in an Excel document; a sample table from this document is included in Appendix 2. The exercises were organised according to the following categories:

- Sets of exercises
- Fast and slow tempos
- Group conditions for high tempos

Primary tempo measurements were organized according to the following categories:

- Maximum and minimum tempos for cycle length
- Maximum and minimum pulse tempos

- Maximum and minimum note rates for each group condition

Other categories included the following:

- Tempos when subdivision was used
- Note rates of the mental subdivisions
- Starting tempo
- Starting note rate

Tempos in which pulses were grouped into a multiple-pulse tactus were noted for fast tempo exercises, whereas tempos when the tactus was felt on pulse level were noted for slow tempo exercises.

For some exercises I noted tempo markings from open categories, mainly in the X^n:3 set, which included the following:

- Pulse tempos and note rates for a comfortable tempo range
- Tempos in which performance was unstable
- Tempos in which the timing relation between the played rhythm and the pulse was clear

3.6.2 Analysing Results Regarding Tempo Limits

Regarding tempo limits, the exercises were compared in various ways to find tendencies across exercises. Since each exercise represented different conditions, groups of exercises could not be easily compared; the organisation and comparisons of data therefore were primarily data driven. The found tendencies were in most cases qualified with reference to experiences from the relevant exercises. These included references to the motions involved, experienced fatigue or tiredness, and good flow.

Three primary tempo measurements were considered and found to be appropriate for different purposes:

- In fast tempos, note rate was found to be the best measurement.
- In slow tempos, pulse tempo found to be the best measurement.
- Cycle length was not found to be a helpful measurement.

Although cycle length could be used to compare exercises with comparable fastest rhythms across different sets, note rates filled the same function.

For present purposes, 40bpm was set as a reasonable lower limit for practically usable pulse tempos, following the tactus tempo range proposed by Lerdahl & Jackendoff (1983, p. 73).[37] Although differences in exercises playable below 40bpm would not be relevant in a musical situation, variations in the results for tempos lower than 40bpm could still be interesting for other purposes and were compared across groups of exercises in the X^n:3 set according to:

- The density of the faster rhythm
- The grouping structure in exercises of equal density
- The framing rhythm

The two latter comparisons are only valid by assuming that density does not affect the lowest playable tempos. Because the same references were used for all compared exercises, it was assumed that any influence they may have exerted would be cancelled out. In addition, exercises were also grouped and compared according to the configuration of the reference, as well as the reference's function in the exercise.

3.6.3 Analysing Results Regarding Structure

Results referring to performance strategies were grouped into categories and compared according to which strategies were used with a specific type of exercise and in which tempo they were helpful. The strategies included the following:

- Motion-based strategies, which included a focus on hand motion, a focus on the downbeat (either every beat or the cycle), continuity of motion, and ways of creating a stable sense of pulse through motion
- Beat-division strategies, which involved mentally organising several pulses into a multiple-pulse tactus, usually on the cycle level
- Composite-pattern strategies, which involved feeling a pulse level tactus and timing the polyrhythm according to the composite pattern of the played rhythm and the pulse
- Subdivision strategies, which involved creating a mental common subdivision which matched either the pulse or the two rhythms in the exercise

[37] This also corresponds to the slowest pulse train that participants followed in Handel (1984, p. 473) and the cycle length that Fraisse (1982 as cited in Grieshaber, 1990, p. 13) found to be unstable (see section 1.2.2).

Exercises in which I noted comfortable tempos, clear timing relations between the played rhythm and the pulse, and timing difficulties (untight or rubato) were also compared within each category.

4. Results

In this section, the results of the practical study are presented as tempo measurements, descriptions of the strategies used, and experiences from the practice sessions. For the tempo limits, common tendencies were searched for across exercises. Regarding the polyrhythmic structure, some aspects were consistent across exercises, but some were specific for certain exercises. Relevant exercises are therefore presented as cases and in-depth examples of how the practice sessions were conducted.

The results regarding tempo limits are presented either as pulse tempos or note rates. Note rates always refer to the fastest rhythm in the exercise, e.g. the 3:1 rhythm in 2:1–3:1.[38] Note rates were found to give more comparable results for the fast tempo exercises, whereas pulse tempos were better for the slow tempo exercises.

4.1 The X:1 Set

4.1.1 Fast Tempo Exercises: Tempo Limits

The results of the fast X:1 exercises are presented with one-, two- and three-group exercises as separate categories. For the one-group condition, the results of the 1:1 exercises proved to be a special case, exhibiting the highest note rates. Because of their special significance regarding motion in playing, these results are presented separately before the other X:1 exercises.

One-Group 1:1 Exercises. When arranged according to the fastest rhythm in ascending order, the highest note rates for the 1:1 exercises follow a curved line with a peak at a 1100bpm note rate for the 1:1–5:1 exercise (Figure 46). The curve could be explained by the sticking patterns used to play the exercises in combination with the "Moeller stroke" technique, which utilises the rebound of the sticks and a whipping motion of the arm to play three strokes in a single motion (ArtOfDrumming, 2019; DRUMMERWORLD, 2020). This technique allows for very fast playing, with a "sweet spot" of three strokes per motion cycle. As shown in the sticking patterns in Figure 47, the 5:1–1:1 exercise consists of three strokes per hand followed by the relatively long quarter note, which can be used to prepare the next flurry of strokes. In the faster subdivisions, there are too many strokes and in the slower are too few for the motion to be most relaxed, which lowers the top speed. This was especially

[38] The relation between the tempo measurements is described in section 3.1.4.

notable in the 1:1–2:1 exercise, in which the hand-to-hand motion was almost continuous, and the relaxing function of the long note was negated.

Figure 46

Results of the one-group 1:1 exercises.

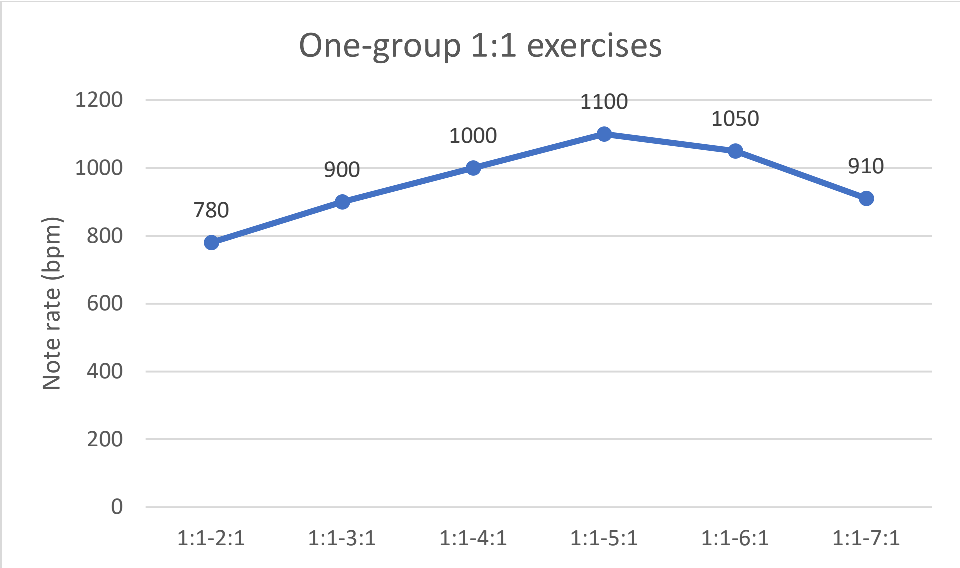
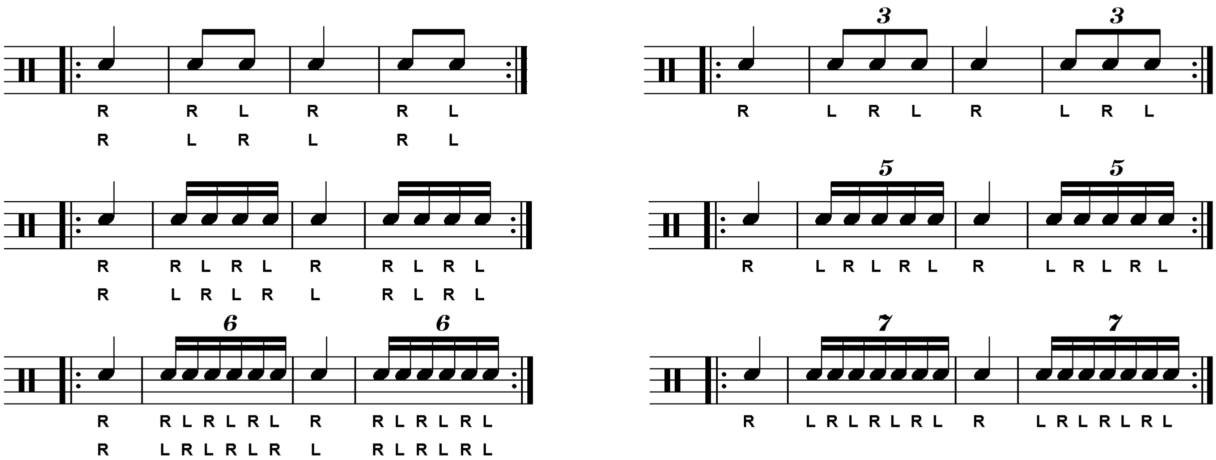


Figure 47

Sticking patterns for one-group 1:1 exercises.



Note. In this and following figures, R signifies the right hand, and L signifies the left hand.

In Figure 47, the exercises with odd subdivisions all have a single sticking indicated. Each group of fast notes starts with the left hand and ends on the quarter note with the right hand. The motion from left to right gives the rhythms a circular continuous feeling that prompts a circular motion when playing. The inversed sticking – starting the groups with the right hand and ending with the left – is equivalent.

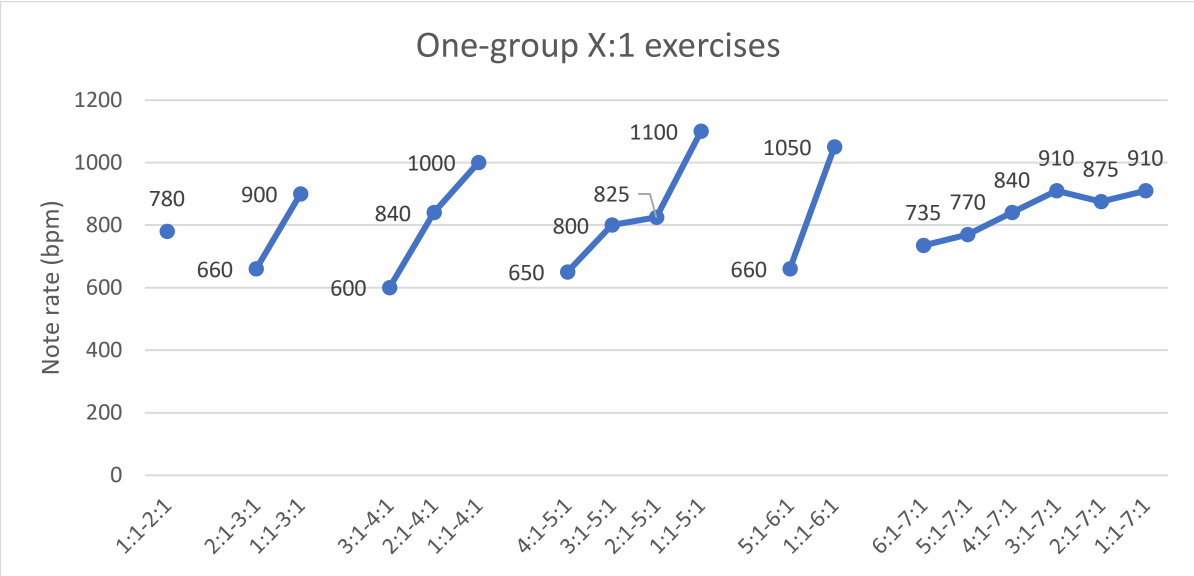
For exercises with even subdivisions, two sticking variants were used. With the first sticking, I always started each beat with the right hand, so that I led with my strong hand. With this sticking, there was slightly less time for preparing the next group of notes, and there was more of a forward-directed motion to the rhythm. The second sticking was strictly alternating, giving slightly more time for preparing the next group of notes. However, the starts and stops within each group were still in the same hand. With this sticking, the motion was less continuously circular than in the odd-numbered subdivisions but was felt rather as a cradling motion of back and forth.

Although the motions and stickings described are not the only possible approaches for playing these rhythms, they show how the configurations allow different ways of playing. In this case, the different subdivisions afford different solutions and motions in performance, based on their odd or even number of strokes.

One-Group X:1 Exercises. Exercises with more differentiated rhythms generally attained higher tempos. The level of *differentiation* corresponds to the relative difference in playing speed of the two rhythms. Separating groups of exercises according to the faster rhythm in the exercise shows the effect of differentiation clearly (Figure 48). For each group of exercises, the maximum tempo increased as the relative difference between the rhythms increased. Exercises with *adjacent-rhythm* combinations, that is, exercises where the faster rhythm had one stroke more than the slower rhythm (2:1–3:1, 3:1–4:1, etc.), attained the lowest maximum tempos. In my experience during the study, the relatively small difference of speed in combination with a fast shift rate blurred the distinction between the rhythms in high tempos, lowering the maximum tempo as a result.

Figure 48

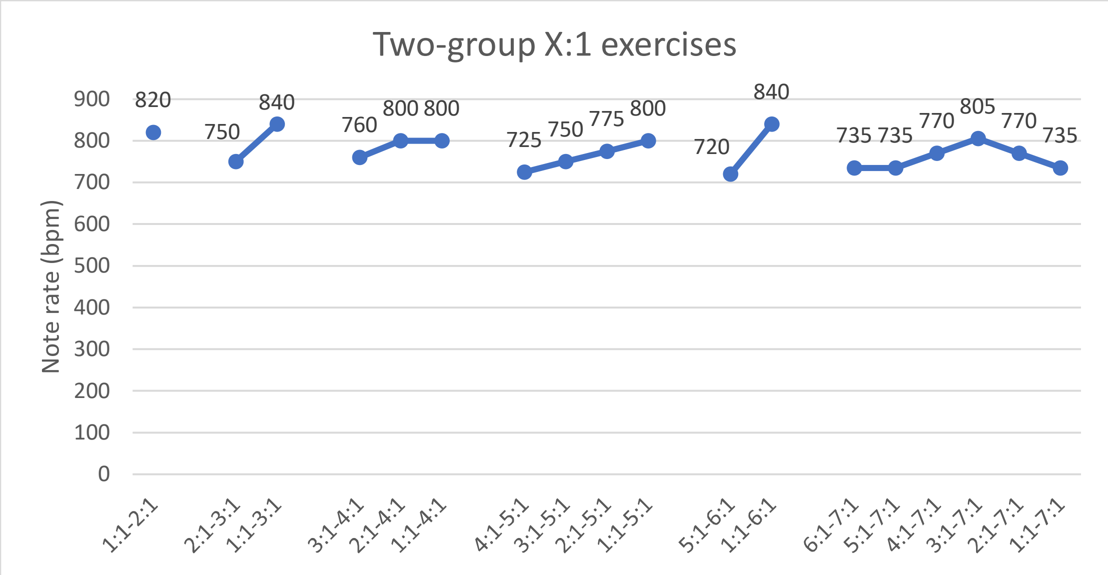
Results of the one-group X:1 exercises.



Two-Group X:1 Exercises. Regarding the two-group exercises, shown in Figure 49, the effect of differentiation was still present but less marked than in the one-group condition. The 1:1-7:1 and 2:1-7:1 exercises are an exception, showing low maximum tempos despite the differentiated speeds. Given the otherwise rather clear tendency and the high tempo attained in the 3:1–7:1 exercise, this discrepancy does not seem too significant. I noted some performance difficulties in the 1:1–7:1 exercise; there was a hitch in my playing around 90bpm (630bpm note rate). On the contrary, while playing the 3:1–7:1, I noted that I had unusually good flow.

Figure 49

Results of the two-group X:1 exercises.

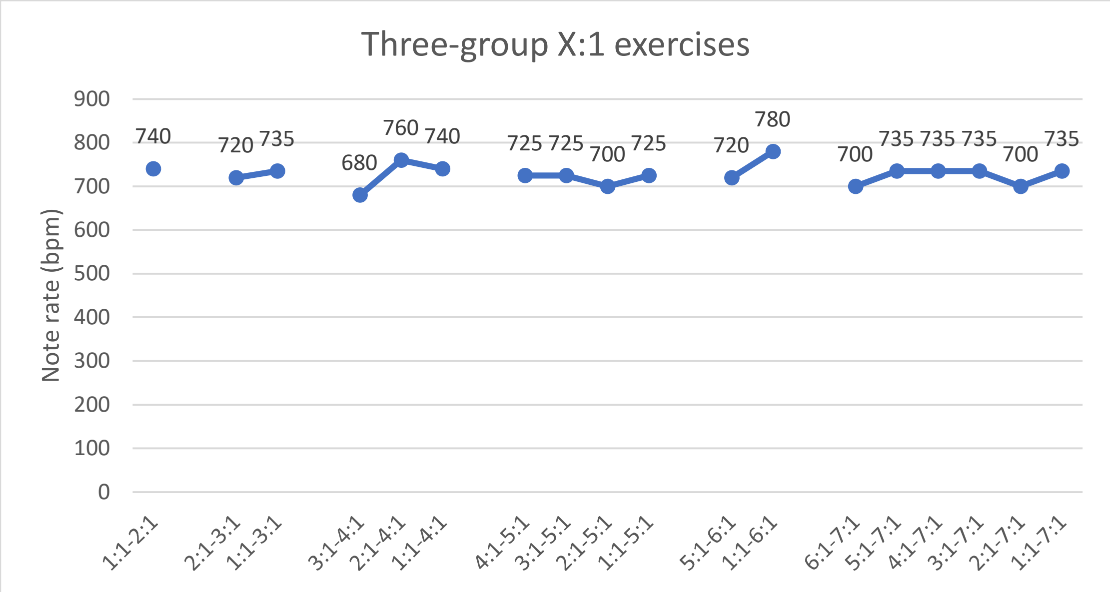


For the three-group exercises (Figure 50), the tempo was generally lower than in the two-group exercises. The attained note rates were consistent enough to be within a reasonable margin of error. The average note rate was around 725–730bpm, and few exercises deviated more than one tempo increment (5bpm pulse tempo) from the average.[39] The repetitions, which formed longer runs of fast notes, evened out the effects of adjacency and differentiation, making overall results more homogenous.

[39] 727bpm mean tempo, 730bpm median tempo. Deviating exercises were 3:1–4:1, 2:1–4:1 and 1:1–6:1.

Figure 50

Results of the three-group X:1 exercises.



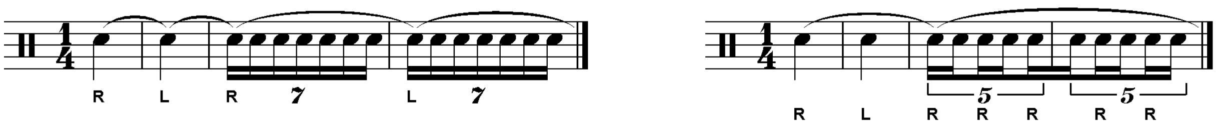
A Final Remark. While constructing the exercises, I assumed that two- and three group exercises could substitute for equivalent one-group exercises, but tested and compared the 2:1–4:1 and 1:1–2:1 exercises regardless. The results from the one-group 2:1–4:1 and the two-group 1:1–2:1 exercises had similar maximum tempos – 840bpm and 820bpm note rates, respectively. These results indicate that the two- and three-group conditions may be comparable to the corresponding one-group exercises, which, since the same note rate corresponds to a higher pulse tempo in the one-group condition, suggests that note rate rather than pulse tempo sets a tempo limit for the X:1 exercises.

4.1.2 Fast Tempo Exercises: Structure

For all exercises played in tempos above 150bpm, pulses were grouped into a two- or three-beat tactus. For the 7:1 exercises, the subdivisions became blurry in note rates around 600bpm and higher. Focusing on the downbeats and the right-to-left motion in the hands was helpful as it created a stable point to guide my playing. On the other hand, the two-group 1:1–5:1 exercise was easier to play in higher tempo ranges while focusing on the right-hand motion. If I focused on each downbeat, the experienced pulse felt somewhat hurried. By focusing on the right-hand motion, the exercise was felt with a two-beat tactus, creating longer phrases and a slower sense of pulse, which allowed for more relaxed playing (see Figure 51).

Figure 51

Sticking patterns focusing on hand motions in fast tempos.



Note. The sticking patterns indicate the focus of my playing. The slurs indicate the sense of pulse.

4.1.3 Slow Exercises

The results for the slow X:1 exercises are shown in Table 8. Although there was some variation in the attained tempos, they were all well below the practically usable tempo range (40bpm). Rather than setting a tempo limit, the configurations in the exercises influenced the choice of strategies for certain tempo ranges.

Table 8

Lowest attained tempos in the X:1 exercises.

X:1	Pulse tempo	X:1	Pulse tempo
1:1–2:1	22.5	**2:1–7:1**	17.5
1:1–3:1	20	**3:1–4:1**	22.5
1:1–4:1	22.5	**3:1–5:1**	25
1:1–5:1	22.5	**3:1–7:1**	30
1:1–6:1	17.5	**4:1–5:1**	22.5
1:1–7:1	22.5	**4:1–7:1**	20
2:1–3:1	27.5	**5:1–6:1**	22.5
2:1–4:1	22.5	**5:1–7:1**	22.5
2:1–5:1	25	**6:1–7:1**	22.5

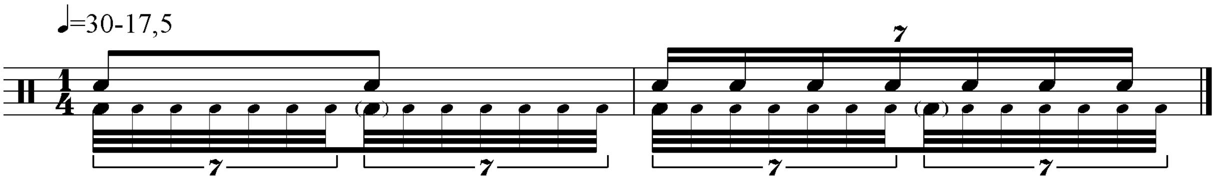
Subdivision Strategies. In tempos above 60bpm, all exercises were played without any particular strategy. In tempos around 60bpm and lower, I subdivided the 1:1 rhythms, matching the other rhythm in the exercise to ensure that I did not rush. The same principle was applied to the 2:1 and 3:1 exercises but in lower tempos. In 35bpm I subdivided the 2:1–

5:1 exercise according to the common subdivision of the two rhythms. The 2:1–7:1 exercise was subdivided in the same way in 30bpm, and the 3:1–4:1 and 3:1–5:1 exercises in 32bpm.

By subdividing, the meter and the timing of the exercises was affected. In the 2:1–7:1 exercise, the 2:1 rhythm was subdivided in 32nd note septuplets, which in effect turned the exercise into the equivalent of four 7/32 measures (Figure 52). The 7:1 rhythm was timed according to the subdivisional grid and had a more of an on- or off-beat feeling than the 7:1 rhythm without subdivisions.

Figure 52

Mental subdivision in the 2:1–7:1 exercise.

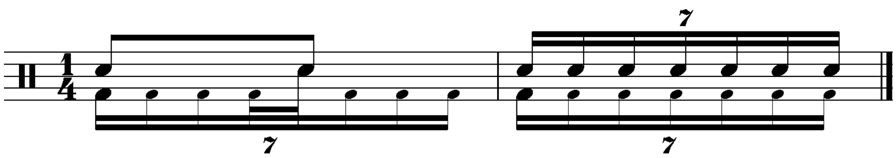


Note. Small notes mark the mental subdivisions, and notes within parenthesis refers to accents in the pattern (i.e. the *added pulses*).

Another approach was to subdivide according to the faster rhythm and play the 2:1 rhythm in between the subdivision (Figure 53). Here, the 2:1 rhythm had a syncopated feeling. The tempos in which these strategies were used were so slow that the subdivisions guided my playing, and the exercises were felt as variants of a seven-type meter.

Figure 53

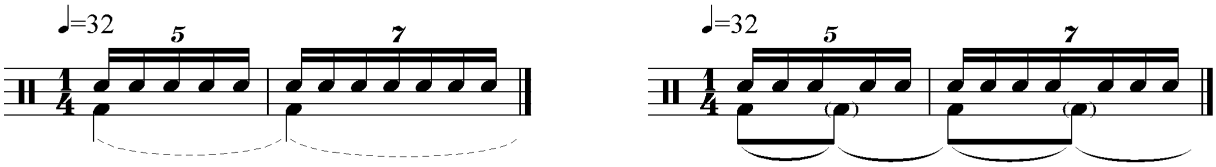
Alternative subdivision in the 2:1–7:1 exercise.



The Added Pulse Strategy. For the more complex exercises in the X:1 category, subdivisions were not helpful since a common grid would be too fast to be usable. Instead, I used an improvised strategy that I call the *added pulse strategy*. I tried to feel an added pulse between the metronome's pulses when the tempo was around 40bpm or lower, in effect creating two-beat cycles. While playing the 5:1–7:1 exercise, for example, the strategy was noted as especially helpful around 32 bpm (Figure 54). It was possible to play without the added pulse, but not with the same stability.

Figure 54

The added-pulse strategy in the 5:1–7:1 exercise.

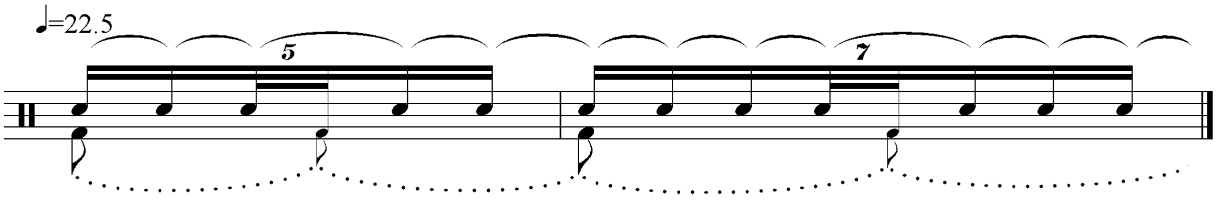


Note. In the figure, a slur indicates a feeling of anticipating the next pulse. The notes within parenthesis represent the added pulses and the dashed slur indicates an unclear sense of duration. The notation is an adapted version of Hasty's (1997) notation of 'projection'.

At 22.5bpm, the added-pulse strategy was no longer helpful. Even if a pulse was added, I could only time it in relation to the played rhythms (5:1 and 7:1) as an inserted fast note (Figure 55). The pulse was simply too slow to be felt – even with an extra pulse, it had lost all feeling of anticipation and prediction (indicated by the dotted line). Nevertheless, the exercise was quite playable.

Figure 55

Limitations of the added-pulse strategy.



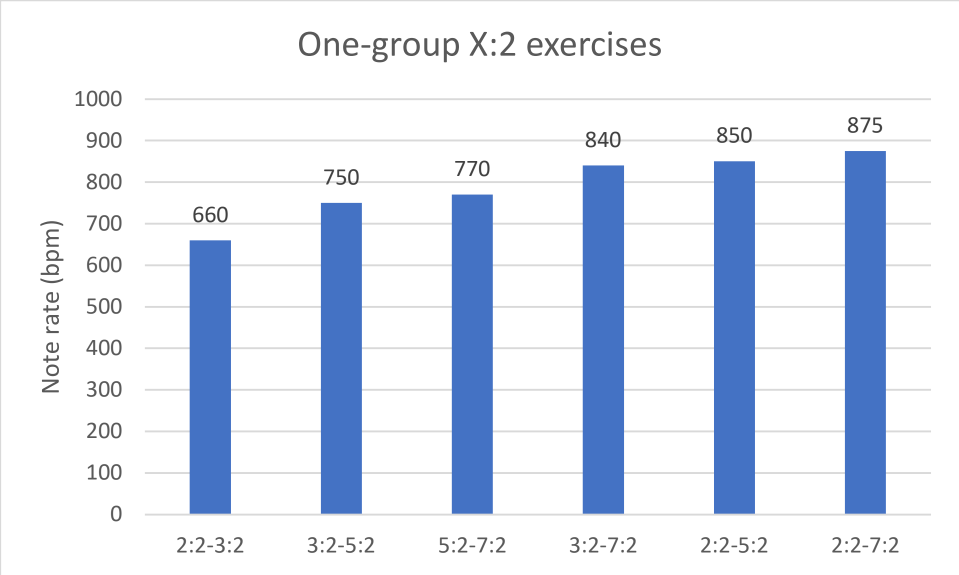
4.2 The X:2 Set

4.2.1 Fast Tempo Exercises: Tempo Limits

One-Group X:2 Exercises. Figure 56 shows the highest attained tempos for the one-group X:2 exercises. The exercises were few, but more differentiated exercises tended to be playable in higher tempos.

Figure 56

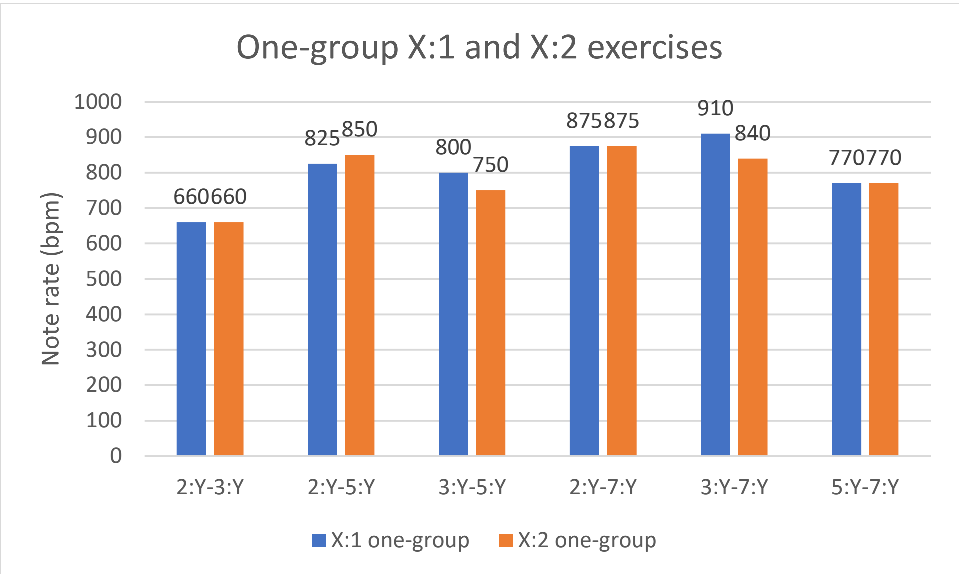
Results for the one-group X:2 exercises, organised according to note rates in ascending order.



The 2:2–3:2 combination, the only adjacent-rhythm exercise in the set, had the lowest playable tempo, 660bpm, the same result as in the one-group 2:1–3:1 exercise (the corresponding exercise in the X:1 set). In fact, for most of the comparable exercises in the X:2 set, I attained tempos close to the X:1 exercises (Figure 57).

Figure 57

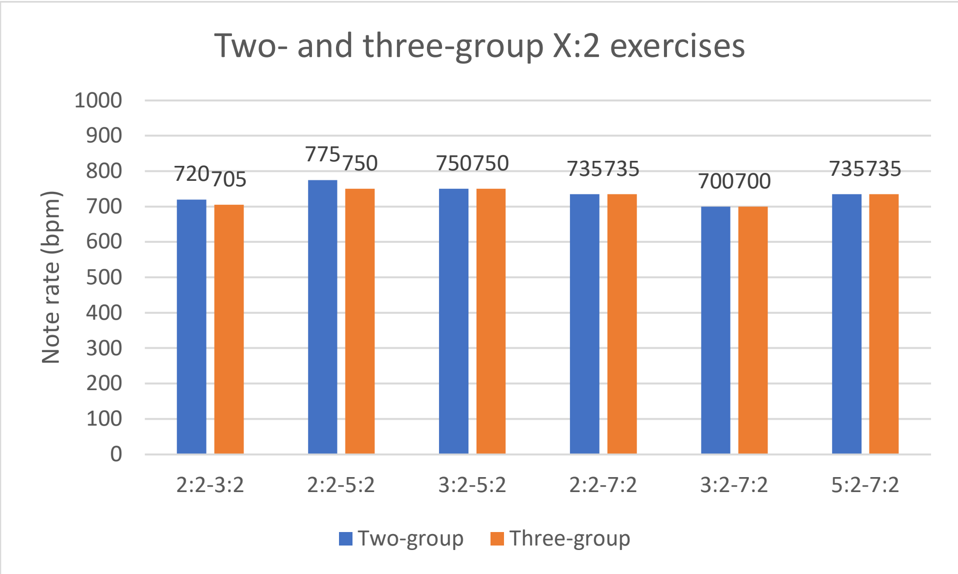
A comparison of the results of the one-group X:1 and X:2 exercises.



Two- and Three-Group X:2 Exercises. The attained tempos in the two and three-group exercises were similar and are presented together (Figure 58). Overall, the results were not highly differentiated and quite comparable to the three-group exercises in the X:1 set. Generally, the tempos attained were lower in the two- and three-group exercises than in the one-group condition. Only the 2:2–3:2 exercise was playable in higher tempos in the two- and three-group condition. The 3:2–7:2 exercises were not practiced over 700bpm note rate due to arm fatigue and pain.

Figure 58

A comparison of the results of the two- and three-group X:2 exercises.



4.2.2 Fast Tempo Exercises: Structure

For all fast tempo exercises, the pulses were grouped into a cycle-level tactus around 130–150bpm. Exercises in which no rhythm was equal to the pulse became more stable when a cycle-level tactus was comfortable, around 140bpm. The 5:2–7:2 exercise benefited the most from a slow tactus; a cycle-level tactus was comfortable already at 100bpm, but a pulse level tactus was also possible.

At tempos around 100–140bpm, there was a transitional range in which many exercises could be played well with either a pulse- or cycle-level tactus, at least for some part of this range. In higher tempos, the cycle was always better as tactus, and in lower tempos the pulse as tactus was the better choice. Relatively complex rhythms were more stable with a cycle-level tactus in slower tempos.

4.2.3 Slow Tempo Exercises

The results of the slow tempo exercises are presented in Table 9. No notable difference was found between the various X:2 exercises, since all were playable below the practically usable tempo range (40bpm). Although the cycles were twice as long for the X:2 exercises, there was no general effect on lowest pulse tempo compared to the X:1 set.

Table 9

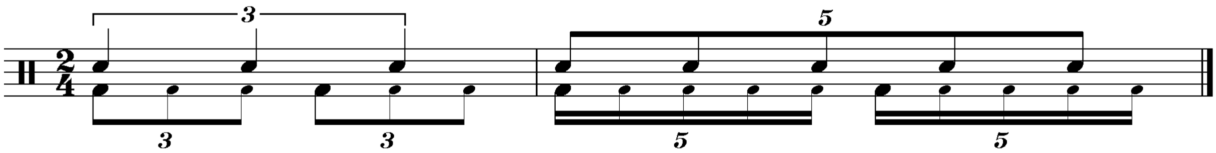
Lowest attained tempos in the X:2 exercises.

X:2	Cycle length (seconds)	Pulse tempo (bpm)
2:2–3:2	6	20
2:2–5:2	6	20
2:2–7:2	6.86	17.5
3:2–5:2	4	30
3:2–7:2	4.36	27.5
5:2–7:2	5.33	22.5

All exercises were subdivided for stability around 45–40bpm. 2:2–3:2 was subdivided for convenience already at 70bpm. The subdivisions were made to match the pulse and not to create a common subdivision for both rhythms. Figure 59 shows the subdivision of the 3:2–5:2 exercise. When the exercises were subdivided, there was a different feeling of flow in the rhythm compared to when it was played as a composite pattern (see section 4.3.2).

Figure 59

Subdivision of the 3:2–5:2 exercise.



4.3 The X^n:2 Set

The X^n:2 exercises were only played in slow tempos, with results similar to the previous slow tempo exercises (Table 10). 5:2–3^5:2 sticks out with a tempo of 35bpm, but all exercises were playable below the practically usable tempo range (40bpm).

Table 10

Lowest attained tempos in X^n:2 exercises.

X^n:2	Cycle length (seconds)	Pulse tempo (bpm)	X^n:2	Cycle length (seconds)	Pulse tempo (bpm)
2:2–3^3:2	6.86	17.5	**5:2–3^5:2**	3.43	35
3:2–3^3:2	6	20	**7:2–3^5:2**	5.33	22.5
5:2–3^3:2	4.36	27.5	**2:2–5^3:2**	5.33	22.5
7:2–3^3:2	4	30	**3:2–5^3:2**	4.36	27.5
2:2–3^5:2	4	30	**5:2–5^3:2**	5.33	22.5
3:2–3^5:2	4.36	27.5	**7:2–5^3:2**	6	20

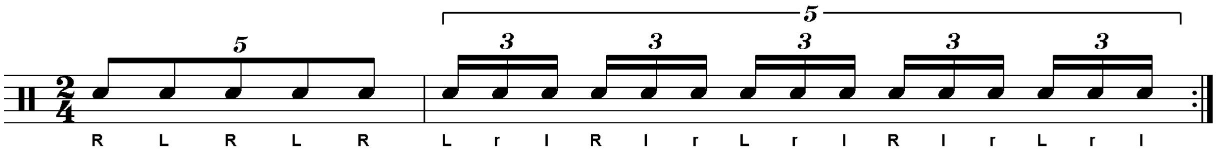
The 3^3:2 exercises were played with a multiple-pulse tactus from the starting tempo of 130bpm to around 100bpm. The other exercises were played with a pulse-level tactus from the start. For all exercises, subdividing the reference to match the pulse was helpful in tempos lower than 60bpm. The more complex rhythms were subdivided in lower tempos (5:2 at 50–35bpm and 7:2 around 35bpm).

4.3.1 Continuity of Motion in X^n:2 Exercises

In many exercises, keeping a continuous motion throughout the exercise helped my timing to be more even. This strategy was used in exercises in which the reference was congruent with the framing rhythm and worked well when the motion from right to left hand was consistent in both rhythms. For example, in the 5:2–5^3:2 exercise (Figure 60), I could focus my attention to the alteration of the hands in the 5:2 measure, using big motions with my whole arm and feeling the energy transfer from right to left and back again. In the shift to 5^3:2, I strived to maintain the same back and forth feeling between the hands (upper-case sticking), while adding the embedded subdivision within this larger motion (lower-case sticking).

Figure 60

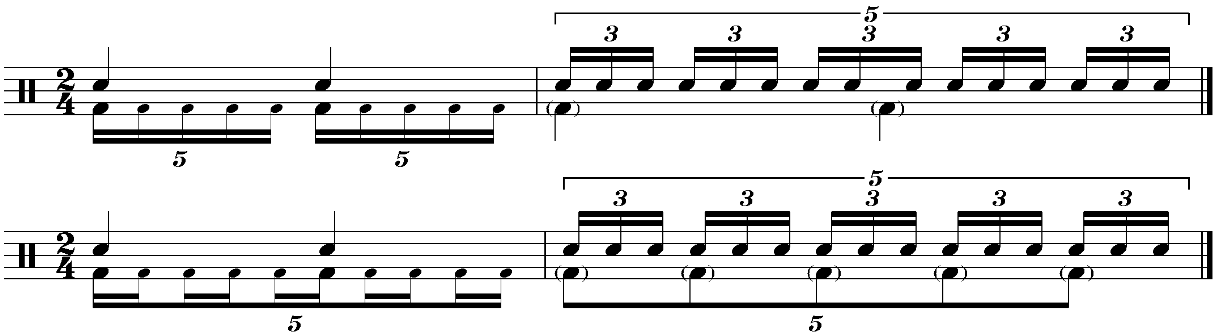
Hand motions in the 5:2–5^3:2 exercise.



The same continuity of motion was helpful even when the movement was not physically carried out but only thought mentally. If I subdivided the reference in the 2:2–5:2 exercise and grouped the subdivision according to the pulse, an accent pattern on quarter note level was created (Figure 61, first system). Even if this pattern matched the pulse, the accent pattern did not support the shift to the next measure. However, if the subdivision was grouped according to the framing rhythm in the following measure (Figure 61, second system), the accent pattern prepared for the motion much in the same way as in the 5:2–5^3:2 exercise, stabilising the exercise.

Figure 61

The implied motions of two subdivision strategies in the 2:2–5^3:2 exercise.



4.3.2 Subdividing the 3^3:2 Exercises.

The following section presents the strategies used for the 2:2–3^3:2 exercise as an example, focusing on how different ways of subdividing the exercises changed the structure of the exercises.

I started practicing the 2:2–3^3:2 exercise at 130bpm. At this tempo it was easiest to feel the tactus on cycle level, aiming for the next downbeat. At 110bpm, it was possible to feel the rhythm with either a cycle-level or a pulse-level tactus. Around 100–90bpm, it was more comfortable to feel the rhythm on pulse level.

At 50bpm, subdividing the reference into triplets improved my timing, but the tendency to spontaneously subdivide the reference was clear already from at least 80bpm. When the tempo was 45bpm or lower, it was possible to feel the 3^3:2 rhythm as a sort of 3:2 pattern against the triplet subdivision (Figure 62, first system). The triplet subdivision was maintained in the shift, and the triplet subdivision in combination with the 3^3:2 rhythm transformed the perceived meter into a kind of 9/8 feeling (Figure 62, second system). Because the framing rhythm guided the timing of the 3^3:2 rhythm, the experienced pulse was effectively shifted from the quarter notes in the first measure to the framing rhythm in the second measure. I heard the pulse from the metronome mostly as a passing moment in the middle of the pattern. Despite the elaborate conception of the rhythm, I found this strategy to be helpful for timing both rhythms in the exercise.

Figure 62

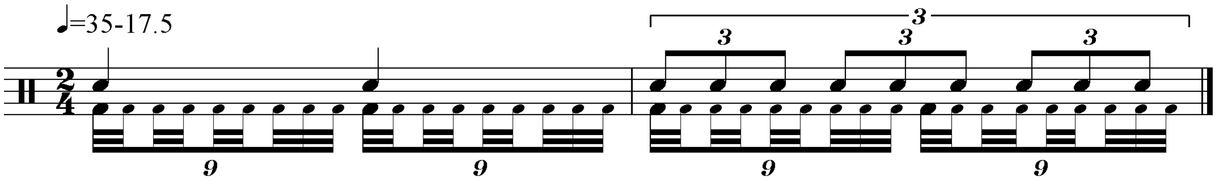
Triplet subdivisions in the 2:2–3:2 exercise.

In tempos around 35–30bpm, the above-mentioned approach became unstable. Instead, I subdivided the pulse in nonuplets, and timed both rhythms according to this subdivision. First, I tried to divide the nonuplets in three-note groups, but even though this division created a stable sense of pulse, the exercise was a bit difficult to play. I therefore tried to group the subdivision in three groups of two and a final group of three notes (Figure 63).[40] With this structure, the exercise was much easier to play.

[40] See Shawn Crowder (2020) for a demonstration of this counting system.

Figure 63

Nonuplet subdivisions in the 2:2–3^3:2 exercise.



Counting the subdivision in this way changed how the rhythm was experienced. First, due to the low tempo, the tactus was felt according to the grouping structure of the nonuplet subdivision, which created an uneven sense of pulse. The played rhythm was then timed in relation to the subdivision. In effect, the 3^3:2 rhythm was no longer felt as such – the feeling of the composite pattern had been replaced by an on- or off-beat feeling, related to the grouping structure of the subdivision.

These two subdivision strategies differed from each other in important ways. Although both strategies could be said to have the same lowest beat level (in Lerdahl & Jackendoff's [1983] sense), this level was not experienced in the same way. In the first strategy, the timing was tied to the framing rhythm. The lowest level, implied by the composite pattern of the triplet subdivision and the 3^3:2 rhythm (the sixteenth notes in Figure 62), was felt as a subsidiary part of this pattern. If I tried to count the subdivision of this lowest level consistently, the quality of my playing suffered. Furthermore, the experienced pulse differed between the two rhythms in the exercise.

In the second strategy, the lowest beat level (the nonuplet subdivision) formed the primary rhythmic layer. The sounding rhythm was played with a timing and phrasing formed in relation to the nonuplet subdivision. Since the groups of the nonuplets marked the tactus, the pulse was still present and consistent throughout the exercise, but its status was displaced to mark a "measure" rather than a pulse proper.

The two strategies had distinct effects on the timing and phrasing of the rhythm, as well as on the experienced pulse. Furthermore, the strategies were helpful in different tempo ranges. The first strategy worked best in the tempo range of 45–35bpm, whereas the second strategy worked best in tempos lower than 35bpm.

4.4 The X:3 Set

In the X:3 set, only three exercises were played in high tempos. The exercises were practiced twice, and the two sessions showed inconsistent results. The fast tempo exercises are therefore discussed in section 4.8.

4.4.1 X:3 Slow tempo exercises

Regarding the slow tempo exercises, the attained tempos were still well below the practically usable tempo range (40bpm) and largely congruent with the results from previous sets (Table 11). As in the X:2 exercises, subdividing the rhythms to match the pulse was helpful in tempos around 50bpm to 35bpm; more complex rhythms were subdivided in lower tempos. A pulse-level tactus was used for all exercises in the set around 100bpm.

Table 11

Lowest attained tempos in the X:3 exercises.

X:3	Cycle length (seconds)	Pulse tempo (bpm)	X:3	Cycle length (seconds)	Pulse tempo (bpm)
3:3–2:3	9	20	**2:3–5:3**	9	20
3:3–4:3	8	22.5	**2:3–7:3**	9	20
3:3–5:3	9	20	**4:3–5:3**	9	20
3:3–7:3	9	20	**4:3–7:3**	7.2	25
2:3–4:3	8	22.5	**5:3–7:3**	8	22.5

4.5 The X^n:3 Set

The X^n:3 set was by far the most numerous, comprising a total of 44 exercises. For a better overview, the results are separated into two tables, with the 2^n:3 and 4^n:3 exercises in Table 12 and the 5^n:3 and 7^n:3 exercises in Table 13. All exercises were playable below the practically usable tempo range (40bpm).

Table 12

Lowest attained tempos in the 2^n:3 and 4^n:3 exercises.

X^n:3	Cycle length (seconds)	Pulse tempo (bpm)	X^n:3	Cycle length (seconds)	Pulse tempo (bpm)
3:3–2^4/4^2:3	6	30	**3:3–4^4:3**	7.2	25
2:3–2^4/4^2:3	8	22.5	**4:3–4^4:3**	6	30
5:3–2^4/4^2:3	4.86	37	**5:3–4^4:3**	6	30
7:3–2^4/4^2:3	6.55	27.5	**7:3–4^4:3**	6	30
3:3–2^5:3	4.5	40	**3:3–4^5:3**	7.2	25
2:3–2^5:3	5.14	35	**4:3–4^5:3**	8	22.5
5:3–2^5:3	4.86	37	**5:3–4^5:3**	5.63	32
7:3–2^5:3	6	30	**7:3–4^5:3**	6.55	27.5
3:3–2^7:3	6	30	**3:3–4^7:3**	6	30
2:3–2^7:3	6.55	27.5	**4:3–4^7:3**	6.55	27.5
5:3–2^7:3	5.14	35	**5:3–4^7:3**	6	30
7:3–2^7:3	5.63	32	**7:3–4^7:3**	6	30

Table 13

Lowest attained tempos in the 5^n:3 and 7^n:3 exercises.

X^n:3	Cycle length (seconds)	Pulse tempo (bpm)	X^n:3	Cycle length (seconds)	Pulse tempo (bpm)
3:3–5^2:3	9	20	**5:3–5^5:3**	6	30
4:3–5^2:3	6	30	**7:3–5^5:3**	5.63	32
5:3–5^2:3	10	17.5	**3:3–7^2:3**	7	27.5
7:3–5^2:3	9	20	**4:3–7^2:3**	7	25
3:3–5^4:3	6.55	27.5	**5:3–7^2:3**	6	30
4:3–5^4:3	7	25	**7:3–7^2:3**	7	25
5:3–5^4:3	8	22.5	**3:3–7^4:3**	6.55	27.5
7:3–5^4:3	6.55	27.5	**4:3–7^4:3**	7.2	25
3:3–5^5:3	6	30	**5:3–7^4:3**	6	30
4:3–5^5:3	6.55	27.5	**7:3–7^4:3**	7.2	25

I usually shifted from a cycle- to a pulse-level tactus in the tempo range of 110–90bpm. Faster references were felt on a pulse-level tactus in lower tempos. Exercises with a $2:3–2^n:3$ combination were comfortable with a pulse-level tactus in 120bpm. When the pulse tempo approached 40bpm and lower, the exercises were often difficult to time well and played with some rubato.

As in the $X^n:2$ set, references congruent with the framing rhythm were easier to time well, especially when a left-to-right hand motion could be preserved. I also noted that body motions synchronised with the pulse could improve my timing in slow tempos. For the $4:3–7^2:3$ exercise, this strategy was used around 50bpm.

Subdivision was less frequently used in these exercises, least often in exercises with noncongruent references; only in the $7:3–5^2:3$, $5:3–2^5:3$ and $5:3–4^4:3$ exercises were the references subdivided to match the pulse. The $7:3–5^2:3$ exercise was subdivided in tempos at 32bpm and below, but it was noted as a poor strategy which was difficult to time well. For the $5:3–2^5:3$ exercise, the subdivisions were similarly disturbing to the $2^5:3$ rhythm. For the $5:3–4^4:3$ exercise on the other hand, subdivision was an effective strategy at 37bpm and lower.

When the 7:3 rhythm was used as a reference, around half of the exercises required a few rounds of practice or were difficult to time well in the start of the practice session before working well.[41] In the tempo range of 70–50bpm in exercises with a 7:3 reference, I noted that the timing relation between the played rhythm and the pulse in the reference was clear.

4.5.1 The Case of $3:3–5^4:3$

The following section presents the $3:3–5^4:3$ exercise as a case that provides an in-depth example of a practice session. The focus is on the experiences that emerged during the practice session and how the timing of the rhythm was experienced in different tempos.

I started practicing the $3:3–5^4:3$ exercise at 90bpm, a rather fast tempo, with a note rate of 600bpm. From the beginning, the rhythm was felt with a pulse-level tactus, which was well established by the 3:3 reference. I timed the 5:3 framing rhythm in relation to the

---

[41] Noted exercises and starting tempos were $2^5:3$ (140bpm), $4^7:3$ (60bpm), $5^2:3$ (140bpm), $5^4:3$ (90bpm), $5^5:3$ (70bpm), $7^2:2$ (120bpm), $7^4:3$ (60bpm). The specific tempo does not seem to have influenced the feeling of stability in these cases.

3:3 rhythm; the physical experience of shifting from three to five strokes over the same time span, which had been practiced and internalised through many other exercises, contributed to a stable performance of the exercise.

70bpm was a rather comfortable tempo to play; the 5^4:3 rhythm was no longer too fast, and the timing relation between the framing rhythm and the pulse could be felt quite clearly. The same sensation was present down to 50bpm. Even though the time-span of each cycle became longer, the relation between the played rhythm and the pulse was clearer. It was easy to feel if my timing was a bit off, and I could adjust my playing rather smoothly.

In 40bpm, I slipped a bit on my first repetition of the exercise. I started playing much too slow on the first beat of the 5^4:3 measure; consequently, I had to play the rest of the rhythm much faster to find my way the next downbeat without losing a few strokes on the way. Yet I still managed to play the next downbeat acceptably on time. Even though I had missed the timing of the first part of the rhythm, I never lost track of the pulse or the strokes to play. In the most free-floating timing of much too late strokes, I was still connected to the pulse, and the strokes still had direction towards the next downbeat. After a few repetitions, the exercise worked effectively.

In tempos from 37bpm downwards, I noticed a slight tendency to rush the rhythms. If I played all the strokes evenly, I often had to wait a fraction of a second before the start of the next measure. Nevertheless, the rhythm was performable at an acceptable level, granted it was played with some rubato. However, even though the lower tempos were more difficult to play accurately, the low tempos also allowed the rhythm to be played in a more flexible manner using some rubato.

Around 32bpm, I tended to react to the pulse, rather than anticipating it. My timing was guided by an internalised sense of the composite pattern of the 5^4:3 rhythm, focusing on the relation between the framing rhythm and the pulse. Since the pulse was external, I reacted to it as early, on time, or late in relation to its expected position in this pattern, and I adjusted the played rhythm accordingly. The tendency of reacting to rather than anticipating the pulse became even more apparent in tempos below 30bpm. For the 3:3–5^4:3 exercise, 27.5bpm was the lowest tempo I could play with an acceptable performance, though rather untight and with some rubato.

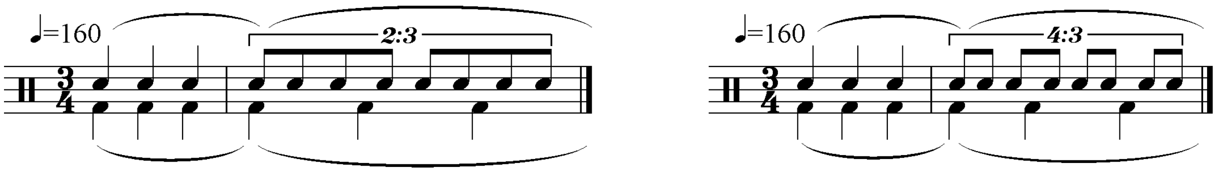
4.5.2 The Cases of 2^4:3 and 4^2:3

Due to the close relation of the two grouping structures of the 8:3 rhythm, I chose to play the 2^4:3 and 4^2:3 exercises as a single exercise. I alternated between playing the 2:3 and 4:3 framing rhythms for each tested tempo and was able to compare them in the playing situation.

When I started practicing in 160bpm, the exercises were felt with a cycle-level tactus. As the timing was controlled from measure to measure, the grouping structure had little effect on timing and stability and only gave a slightly different feeling to the rhythm (Figure 64).

Figure 64

Timing goals for fast tempos in the 3:3–2^4:3 and 3:3–4^2:3 exercises.



In the tempo range of 130bpm to 100bpm, the pulse became increasingly salient. The cycle lengths in these tempos were between 43 and 33bpm, thus difficult to feel over one beat. With an increasingly salient pulse, the relation between the framing rhythm and the pulse increased in importance, and the 2:3 framing rhythm was more stable than the 4:3 framing rhythm.

The more stable 2:3 framing rhythm could be explained by the implied subdivision of the framing rhythm. This subdivision was not explicitly counted, but in order to distinguish the timing relation between the framing rhythm and the pulse, the duration between the onsets should reasonably be long enough to be rhythmically salient (i.e. at least longer than 100ms/600bpm, but preferably slower). For 2:3, the tempo of this implied subdivision would be 260–200bpm. For 4:3, the implied subdivision would be twice as fast (520–400bpm). Reasonably, the longer durations within the upper range of pulse tempos are more salient and therefore easier to play than the shorter durations.

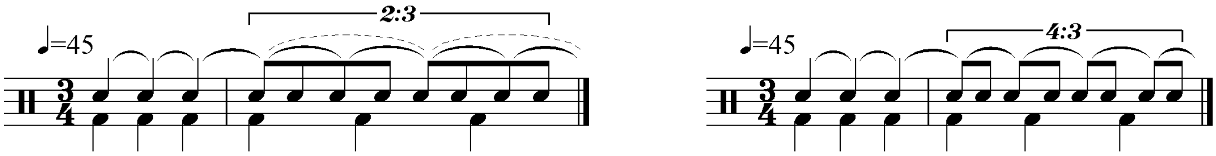
The increased difference in stability could also be explained by the tempo of the framing rhythm. For the 2^4:3 grouping structure, this tempo was between 87 and 67bpm, close to the historical tactus of 70bpm (Lerdahl & Jackendoff, 1983, p. 73).The corresponding tempos for the 4^2:3 grouping structure were 173 to 133 bpm: either above the *temps courts* threshold (indicating that the notes in the 4:3 framing rhythm would tend to group together), or, in the lower tempos, still in the upper range of comfortable pulse tempos.[42]

In lower tempos, the 4^2:3 grouping structure gradually became more stable than the 2^4:3 rhythm. This was clear around 80bpm. In this tempo, the tempos of the framing rhythms were 107bpm for 4^2:3 and 53bpm for 2^4:3, and the tempo of the implied subdivision 320bpm and 160bpm. The duration of the framing rhythm seems to be a better explanation for the difference in stability than the implied subdivision, since 53bpm is on the low side of pulse tempos and 107bpm is close to the "maximal pulse salience" tempo.

In tempos lower than 50bpm, the 2:3 framing rhythm was very unstable. The embedded subdivision was so slow that the feeling of a single four-note group over the framing rhythm was difficult to sustain and tended to be felt as two groups. The two-note groups, indicated with slurs in Figure 65, were more palpable than the longer four-note group, indicated with dashed slurs. Since the grouping structure of the embedded subdivision was split in two, in effect another layer of rhythmic structure was added, and the distinction between 2^4:3 and 4^2:3 became meaningless.

Figure 65

Timing goals for slow tempos in the 3:3–2^4:3 and 3:3–4^2:3 exercises.

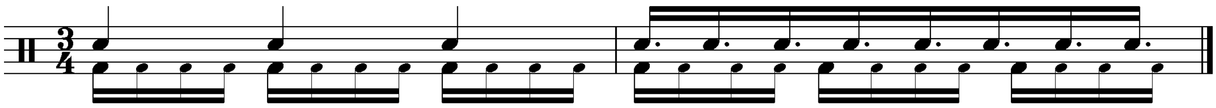


Subdivision strategies were used for the 3:3–2^4/4^2:3 and 2:3–2^4/4^2:3 exercises around 45–40 bpm and lower. I subdivided the beat as sixteenth notes (4:1) and played the rhythm as dotted sixteenth notes over the subdivision (Figure 66). Subdividing 32nd notes throughout the exercise was not helpful in tempos higher than 25bpm.

[42] I am here alluding to the upper tactus limit of 160bpm mentioned in Lerdahl (1983, p. 73).

Figure 66

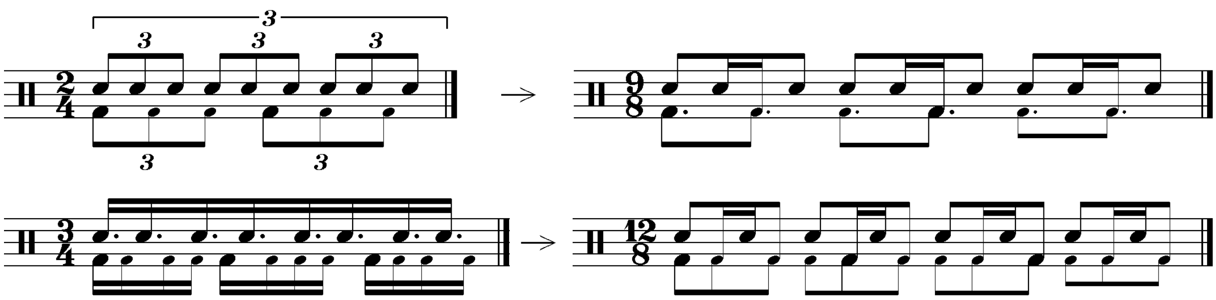
Subdivisions in the 3:3–$2^4/4^2$:3 exercise.



The results of the $2^4/4^2$:3 exercises were somewhat similar to the 3^3:2 exercises; the relation between played rhythm and subdivision forms the same pattern but in a reversed manner (Figure 67). It is also interesting that in the case of 3^3:2, I experienced the pulse as less important in respect to timing, simply a passing moment in the middle of the groups. However, in the 4^2:3 rhythm, the pulse was more present, and the played rhythm felt like a syncopated pattern against the pulse.

Figure 67

Comparison of subdivision strategies in the 3^3:2 and $2^4/4^2$:3 exercises.



4.6 The X:4 Set

Three fast tempo exercises were played in the X:4 set. As in the X:3 set, the results regarding high tempo limits were inconsistent and are discussed in section 4.8.

The slow tempo exercises were playable in tempos comparable to the other sets (Table 14). In the slowest tempos, cycle lengths were extremely long, and the rhythm could only be held together through the subdivision. The greatest challenge for the 4:4 exercises was to keep track of the length of the cycle in the slowest tempos and not accidentally play three strokes before shifting to the other rhythm.

Table 14

Lowest attained tempos in the X:4 exercises.

X:4	Cycle length (seconds)	Pulse tempo (bpm)
4:4–3:4	13.71	17.5
4:4–5:4	12	20
4:4–7:4	8	30
3:4–5:4	9.6	25
3:4–7:4	8	30
5:4–7:4	8	30

The tactus was felt on pulse level in tempos between 130 and 80bpm. Generally, the tactus was felt on pulse level in lower tempos for more complex rhythms compared to simpler configurations. In tempos above 130bpm, pulses were grouped into a two-beat tactus. In the fast tempo exercises, pulses grouped into a cycle-level tactus (four beats) between 180 and 260bpm. Subdivision was generally possible in tempos around 60–45 bpm, becoming more helpful in lower tempos. For instance, in the 3:4–7:4 exercise, I noted that the exercise was playable without subdivisions in 37bpm, but my timing was unstable. When subdividing the 7:4 rhythm, it was better not to focus on where the strokes were to be played in the subdivision (i.e. the numerical position on the subdivisional grid) but rather just count to keep time and let the movement guide the strokes.

4.7 Comparing Exercises

In this section, the X^n:3 exercises are grouped and compared according to the framing rhythm, the density, and the grouping structure of the tested rhythms. In addition, the references are grouped and compared according to their configurations and their function in the exercises.[43] This type of comparison is not unproblematic, since it assumes that all exercises within each group represent equal conditions. Because only mean values were compared, it is not unlikely that these results are due to chance, especially considering that the compared tempos are extremely low. The comparisons were nevertheless carried out,

[43] The full datasets of the compared groups of exercises are included in Appendix 1, Tables C to I.

since they give insight regarding factors that potentially influence the lowest playable tempos.

Table 15 shows the mean results for the lowest tempos attained in the X^n:3 set, grouped by the framing rhythm of the exercises. The $2^4/4^2$:3 exercises were excluded since they belong to both the 2:3 and 4:3 category. This comparison assumes that the embedded subdivision – and therefore the density – does not affect the lowest playable tempos. The results showed a clear differentiation between the 2^n:3 set and the other exercises. Consistent with these results, the 2:3 framing rhythm was noted as difficult in slow tempos (see section 4.5.2); often, I could not rely on the framing rhythm for timing the exercise but instead focused on the relation between the embedded subdivision and the pulse.

Table 15

Mean tempos of the X^n:3 exercises grouped according to the framing rhythm.

Framing rhythm	Mean pulse tempo (bpm)
2:3	33.31
4:3	28.29
5:3	25.79
7:3	26.88

Table 16 shows the mean tempos of the X^n:3 exercises, grouped according to density. Small differences and large differentiation between the grouping structures would imply that the density affects less than the grouping structure. Differences between the groups were small; only the 20:3 density group (5^4:3 and 4^5:3 exercises) showed a slightly lower mean tempo, thus making the comparison of the framing rhythms more valid.

Table 16

Mean tempos of X^n:3 exercises grouped according to the density of the faster rhythm.

Density group	Mean pulse tempo (bpm)	Density group	Mean pulse tempo (bpm)
8:3	29.25	**20:3**	26.19
10:3	28.69	**25:3**	29.88
14:3	29.00	**28:3**	28.13
16:3	28.75		

By comparing the grouping structures in exercises with the same density (Table 17), the 10:3 and 14:3 density groups showed differentiated results for the two grouping structures, supporting the proposition that the framing rhythm influences the playable tempo range. The difference was less marked in the 28:3 density group and much less marked in the 20:3 density group.

Table 17

Mean tempos of X^n:3 exercises grouped according to the grouping structures of exercises with the same density.

Density: 10:3	Mean tempo	Density: 14:3	Mean tempo	Density: 20:3	Mean tempo	Density: 28:3	Mean tempo
2^5:3	35.50	**2^7:3**	31.13	**4^5:3**	26.75	**4^7:3**	29.38
5^2:3	21.88	**7^2:3**	26.88	**5^4:3**	25.63	**7^4:3**	26.88

By comparing the results grouped by the references in the exercises, the 5:3 references and the noncongruent slow references showed the highest mean tempo (Table 18). The same was found in the X:2 exercises, but the sample of exercises was very small (Table 19). The frame condition had the lowest mean tempo. The exercises were, however, played in the same order (pulse, frame, slow, fast), which could have influenced the attained tempos.

Table 18

Mean tempos of the X^n:3 exercises grouped according to the reference's function and configuration.

References X^n:3	Average tempo	References X^n:3	Average tempo
Pulse	28.41	**3:3 (pulse)**	28.41
Frame	25.91	**2:3/4:3**	27.05
Slow	30.32	**5:3**	30.09
Fast	28.77	**7:3**	27.86

Table 19

Mean tempos of the X^n:2 exercises grouped according to the reference's function and configuration.

References X^n:2	Average tempo	References X^n:2	Average tempo
Pulse	23.33	**2:2 (pulse)**	23.33
Frame	23.33	**3:2**	25
Slow	30	**5:2**	28.33
Fast (7:2)	24.17	**7:2 (fast)**	24.17

4.8 Replayed Exercises

4.8.1 X:1 and X:2 Exercises

To test the reliability of the results, a sample of exercises were replayed. This was done more than two months after I had finished the first practice sessions. During these sessions, the exercises were video recorded. For the X:1 exercises, only the exercises with the highest and lowest attained tempos from the first session were tested; the group conditions were treated separately. These exercises were chosen based on the assumption that these results would be most likely to change when replayed. For the X:2 exercises, all exercises were replayed, randomly assigned to one of the group conditions. All X:3 and X:4 exercises were replayed in the one- and two-group conditions. For most replayed X:1 and X:2 exercises, the results were consistent with the first practice session (Tables 20 and 21). The only exercises deviating more than one tempo increment were the 3:1–4:1 and 1:1–3:1 exercises.

However, the fast X:3 and X:4 exercises showed substantial variation from previous results and are here discussed in detail.

Table 20

Note rates of the original and replayed X:1 exercises.

One-group exercises	Note rate	Two-group exercises	Note rate	Three-group exercises	Note rate
3:1–4:1	600	**1:1–3:1**	840	**3:1–4:1**	680
3:1–4:1 replayed	680	**1:1–3:1 replayed**	900	**3:1–4:1 replayed**	700
1:1–5:1	1100	**5:1–6:1**	720	**1:1–6:1**	780
1:1–5:1 replayed	1100	**5:1–6:1 replayed**	720	**1:1–6:1 replayed**	780

Table 21

Note rates of the original and replayed X:2 exercises.

One-group exercises	Note rate	Two-group exercises	Note rate	Three-group exercises	Note rate
2:2–3:2	660	**2:2–5:2**	775	**3:2–7:2**	700
2:2–3:2 replayed	660	**2:2–5:2 replayed**	800	**3:2–7:2 replayed**	735
3:2–5:2	750	**5:2–7:2**	735	**2:2–7:2**	735
3:2–5:2 replayed	775	**5:2–7:2 replayed**	735	**2:2–7:2 replayed**	735

4.8.2 X:3 Exercises

In the first practice sessions of the X:3 exercises, pulses were grouped into a cycle-level tactus at 150 or 160bpm. The 2:3–4:3 exercise was noticeably more difficult and had to be practiced in the tempo range of 180–250bpm. In higher tempos, the exercise proved easier to play.

In the replayed session, the 3:3–4:3 exercise was practiced from 300bpm and was repeated a few times before it worked well. Performance was then unproblematic up to 435bpm (580bpm note rate). After three attempts in 450bpm I gave up on the one group condition, but the two-group condition proved quite playable up to 525bpm (700bpm note rate). In tempos above 400bpm, it was difficult to resynchronise with the metronome's downbeat if I lost track of it (since there was no differentiation between downbeats and pulses in the sound signal), and I often had to restart the metronome to align with it again. The attained tempos in the 3:3–4:3 exercise were much higher than during the first practice session, in which I could play the exercises in 405bpm for the one-group condition and 420 for the two-group condition.

I started recording the 3:3–2:3 exercise at 405bpm, having previously practiced from 300bpm. The one-group condition was uncomfortable and stiff in the tempo range of 300 to 360bpm but felt smoother in 375bpm. In 420 bpm, I had severe problems. My playing was stiff and unsynchronised, and after four minutes of bad playing, I ended my attempt. During my first session, I played the 3:3–2:3 exercise in 450bpm and 465bpm for the one- and two-group condition respectively – a higher tempo than when I replayed these exercises. As my playing throughout this last session was stiff and generally poor, I decided to try again the next day.

I started the next session in 390bpm. This time, I felt the tactus on two-cycle level (six pulses), which I had not tried the day before. Now my playing was much smoother, with good synchronisation and timing. A few restarts were still required, but the exercise was quite playable to 435bpm. 450bpm almost worked, but only with great effort. Although I could only play a bit faster, the quality of my playing was much improved by the slower sense of pulse.

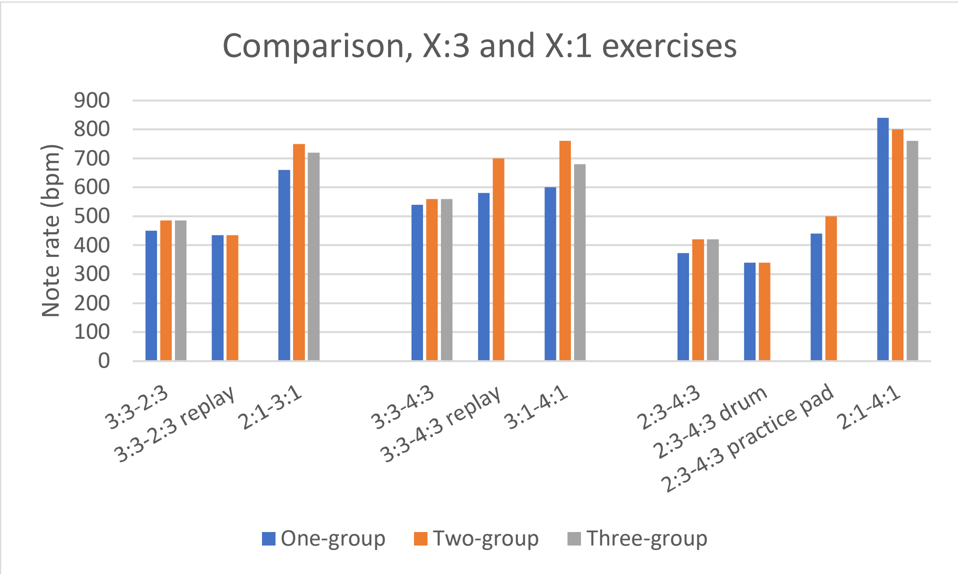
The 2:3–4:3 exercise was started in 120bpm and recorded from 210bpm. The exercise itself was more difficult than others. In addition, I noticed that the 4:3 rhythm proved more difficult than the 2:3 rhythm, contrary to the other two exercises. I managed to play the exercise in 240bpm for the one-group condition and 255bpm for the two-group condition. During my first session, I was able to play in 280bpm and 315bpm for the same group conditions.

While replaying the exercise, it was sometimes difficult to hear the metronome, and it was easier to play in soft dynamics. I therefore did a final test on my practice pad in order to play as softly as possible, which made the exercise much easier. I could clearly hear the metronome and the rhythms could be evenly played without much effort. The one-group condition was playable up to 330bpm (440bpm note rate) and the two-group condition worked up to 375bpm (500bpm note rate) – higher tempos than in the first session.

Figure 68 summarises the results from the X:3 exercises from the replayed and the original sessions.[44] The 2:3–4:3 exercise proved more difficult than the other exercises in both sessions. Feeling a slower tactus improved my playing 3:3–2:3 exercise, and hearing the metronome clearly was important for the 2:3–4:3 exercise. Playing on the snare drum may have affected the results of the 3:3–2:3 exercise but does not seem to have influenced the 3:3–4:3 results to any large extent.

Figure 68

A comparison of the results of the X:3 and X:1 exercises.



4.8.2 X:4 Exercises

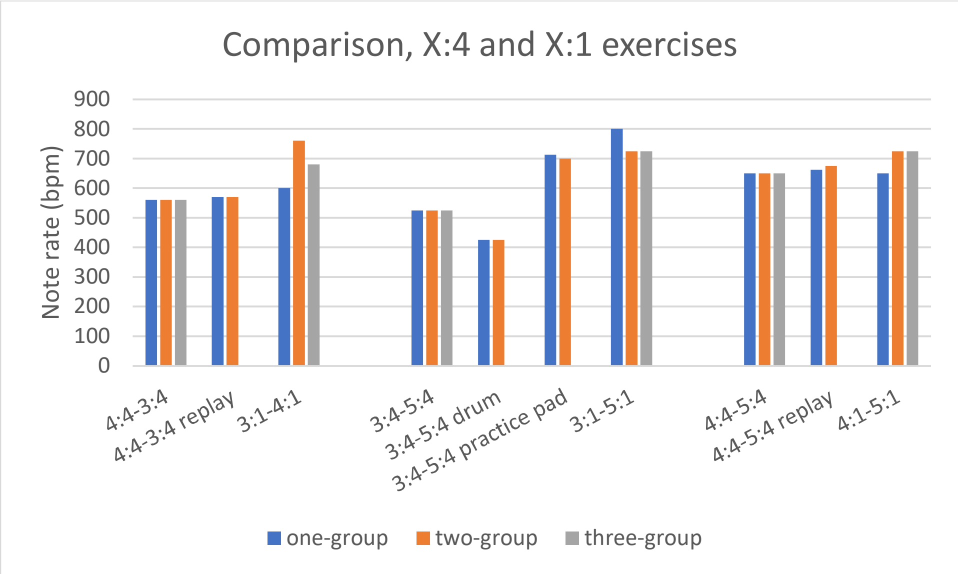
Figure 69 shows the attained tempos in the X:4 set for the replayed and the original practice sessions.[45] The results of the 4:4–3:4 and the 4:4–5:4 exercises were similar for both sessions. The 3:4–5:4 exercise was difficult, and I had to practice it many times around 260bpm. The 3:4–5:4 exercise was practiced on both a snare drum and a practice pad. Like the 2:3–4:3 exercise, it proved easier to play on the practice pad. However, while the 2:3–4:3

[44] The full dataset is included in Appendix 1, Table J.
[45] The full dataset is included in Appendix 1, Table K.

exercise had a limited tempo range, the attained tempos for the 3:4–5:4 exercise were high, almost the same as in the 3:1–5:1 exercise – and even higher than the 4:4–5:4 exercise.

Figure 69

A comparison of the results of the X:4 and X:1 exercises.



Whether pulse tempo or note rate limited the playable tempo range of the X:4 exercises was not clear. The 4:4–3:4 exercise was clearly limited by pulse tempo; it was simply not possible to follow the metronome in higher tempos than 570bpm. The attained pulse tempos in the replayed 3:4–5:4 exercise were also 560 and 570bpm, but the note rates were close to the 3:1–5:1 exercise. For the replayed 4:4–5:4 exercise, pulse tempos were 530 and 540bpm, and the note rates were similar to the 4:1–5:1 exercise; for the one-group condition, the note rates were even higher in the 4:4–5:4 exercise than in the 4:1–5:1 exercise.

5. Discussion

This chapter first discusses how the polyrhythmic configurations in the study relate to tempo limits and how tempo influences the polyrhythmic structure, with accompanying suggestions for future research.[46] Regarding tempo limits, the X:1 and X:2 exercises showed similar results and are discussed together. The same applies to the X:3 and X:4 exercises. The slow tempo exercises are discussed separately. The discussion of polyrhythmic structure is divided according to motion-based strategies, the strategies related to the grouping of pulses, and the subdivision strategies, concluding with a discussion of the tempo ranges of the performance strategies. Next, several other considerations are discussed, including a comparison of the conditions of fast and slow tempos, hypotheses on factors influencing the "comfortable tempo range," a discussion of problems with relating performance difficulties to the playable tempo range, and a discussion of the implications of these respective topics. The chapter concludes with further detailed suggestions for future studies.

5.1 Tempo Limits

In fast tempos, the polyrhythmic configurations affected the playable tempo range. The combination of rhythms exerted more influence on the playable tempo range than single configurations. Most notably, differentiation, adjacency, and length (the number of notes in the faster rhythm) affected the high tempo limits.[47] The X:2 set showed results comparable to the X:1 set, and the polyrhythmic configuration did not substantially affect tempo limits for X:2 rhythms. For the X:3 and X:4 set, few exercises were tested, and the results were therefore inconclusive.

The exercises were played from a medium range tempo to both high and low extremes. This approach may have influenced the results by priming the performance through extended practice during the sessions. For the high tempos, the repetitions ensured that I was well warmed up in the highest tempos. In low tempos, the repetitions ensured that the pattern was well-rehearsed in the slowest tempos. This approach was an active choice, as it reduced the risk of stiffness or lack of practice negatively influencing the results.[48]

[46] Key terms are presented in sections 2.4, 3.1, and 3.2.
[47] Differentiation and adjacency are described in section 4.1.1.
[48] In a concert situation, where such priming is not possible, the playable tempo range is likely to be lower.

While increasing fatigue hindered me from practicing all fast tempo exercises in the X:3 and X:4 sets, the individual exercises did not seem to be negatively influenced to a substantial degree. Some exercises were indeed affected: the two- and three-group conditions of the 3:2–7:2 exercise were not practiced over 700bpm for this reason. However, considering that the replayed sessions of the X:1 and X:2 exercises, which were played when my arms had recovered, showed mostly consistent results, any possible influence of fatigue does not contradict the general tendencies of the results.

5.1.1 X:1 and X:2

In the X:1 set, the tempo limit was largely determined by the degree of differentiation between the two rhythms in the exercise. The most differentiated exercises in the X:1 set, in which one rhythm matched the pulse (the 1:1 exercises), showed the highest attained tempos of all exercises. On the other end, adjacent-rhythm exercises, in which only one stroke differed between the two rhythms in the exercise, showed the lowest attained tempos in the X:1 set.

The effect of differentiation was related to the motions involved in playing. A relaxed and continuous motion followed by relative rest allowed for faster playing, while longer spans of fast notes were more strenuous to play, thus lowering the maximum attainable tempo. The effect of differentiation was therefore most clear in short exercises. However, since the movements were idiosyncratic to myself and the instrument, this effect is likely context dependent.

The effect of adjacency was also most notable in the shorter exercises. The quick alternation between the rhythms lowered the tempo, since the mind or the hands could not keep up with the quick changes. Therefore, longer exercises with slower alternation were playable in higher maximum tempos.

The length of the exercises thus had a levelling effect. For both the X:1 and the X:2 set, the three-group exercises showed rather consistent tempo limits. This would indicate that long runs of fast notes have a consistent maximum tempo. The tempo limits in the two-group exercises were more varied than in the three-group condition for the X:1 set but more consistent in the X:2 set. In the one-group condition, the configurations and combinations had a larger effect, with highly varied tempos attained in both the X:1 and the X:2 set.

For the X:2 set, the polyrhythmic configurations did not seem to substantially affect the highest attainable tempo, as the results for the X:1 and the X:2 set were very similar. It is therefore likely that, had combinations of X:2 rhythms and non-polyrhythmic configurations been tested, the effects on tempo limits found in the X:1 set would carry over to other X:2 rhythms.

The similar results for the X:1 and X:2 sets, together with the consistent note rates in the three-group exercises, indicate that note rates set the tempo limits for rhythms with one- and two-beat cycles. Even if the effects of adjacency and differentiation was tied to short exercises, shift rate did not correlate to attained tempos: for relevant adjacent-rhythm exercises, the shift rates varied from 220bpm to 110bpm, but the note rates were between 600bpm and 660bpm. Similarly, the physical motions of playing – rather than length of the rhythms – likely allowed the high tempos to be attained in the most differentiated exercises.

The effects of adjacency and differentiation seem to be a promising explanation for performance limits but unfortunately could not be thoroughly tested for other sets than X:1. I had not anticipated these effects while designing the exercises and had excluded most of the exercises that would have been relevant for testing these effects. The effects of adjacency and differentiation therefore represent a highly interesting topic for a future study.

5.1.2 X:3 and X:4

For the X:3 and X:4 exercises, the polyrhythmic configurations in these sets influenced performance, but the results were inconclusive due to few exercises and varying tempos for the two practice sessions.

The pulse tempo limited the tempo range for some of the X:3 and X:4 exercises, since I often lost track of the metronome's pulse groupings. For the X:4 exercises, the highest attained pulse tempo was 570bpm; higher pulse tempos could not be followed even without playing. However, because the note rates in some exercises were similar to the attained tempos for comparable X:1 exercises, the tempo range may also have been limited by the note rate.

Handel (1984, p. 473) proposed that pulse trains in polyrhythms were difficult to follow in tempos faster than 300bpm (200ms) but possible to follow down to 500bpm (120ms). This duration limit corresponded to some extent to the highest attained pulse tempos in the X:3 and X:4 sets, which ranged from 255bpm (2:3–4:3 replay, drum) to 570bpm (4:4–3:4 replay, 3:4–5:4 replay, practice pad). It is therefore possible that single-line polyrhythms are similarly constrained in situations when the pulse tempo is high enough that listening, rather than playing, sets the limit. However, whether pulse tempo or note rate limited performance was not always clear for these exercises. This issue requires further examination of polyrhythms spanning three or more pulses before even preliminary conclusions can be drawn.

While I played most exercises on a practice pad, some exercises were played on a snare drum in a practice room. The headphones used for the snare drum sessions let through some sound from the snare drum, which sometimes made the metronome's clicks unclear. For the 2:3–4:3 and 3:4–5:4 exercises, playing on a practice pad was important for reaching high tempos, indicating that the clarity of the metronome's sound can influence performance. For most exercises, however, the acoustic environment does not seem to have substantially influenced the results, since all other exercises were playable during the replayed sessions in high tempos on a snare drum with the same conditions as the 2:3–4:3 and 3:4–5:4 exercises. Clarity of metronome sound may therefore be most significant for exercises in which neither rhythm matches the pulse and in which pulse tempos are very high.

5.1.3 Slow Tempos

In slow tempos, all exercises were playable below the practically usable tempo range (40bpm). Thus, no practical difference was found between the various configurations. As the cycle length of the longest exercises spanned around nine to twelve seconds, even beyond the extent of the perceptual present, the length of the cycle did not limit performance for any of the tested exercises. However, all sets were based on the most common meters, which could be easily grouped into even two- or three-beat divisions. It is possible that polyrhythms played against five or more pulses may be limited by the increasingly long spans of time or the more complex pulse groupings.

In these exercises, the maximum cycle length of 1580ms (38bpm) proposed by Fraisse (as cited in Grieshaber, 1990, p. 13) was greatly exceeded. Since the pianists in Bogacz's (2005) study also exceeded Fraisse's maximum cycle lengths by playing polyrhythms over seven second cycles, this result was not surprising. However, the 5:3 rhythm, which Bogacz tested, is easily subdivided in slow tempos, which could explain why the long cycles were playable. However, some of the rhythms in this study did not allow subdivision strategies even in the lowest tempos, which means that in performance, the polyrhythms neither depended on the cycle length nor relied on a common subdivision in long cycles to be playable.

Since most exercises were playable in tempos below the two-second beat perception threshold, pulse tempo per se cannot be said to have limited performance. By analysing the mean tempos for groups of exercises in the X^n:3 set, some aspects that potentially affected the lowest playable tempos were examined. Exercises with the same density showed consistent results across the groups and were deemed not likely to correlate to tempo limits. The slow noncongruent reference, as well as the 5:3 reference, showed slightly higher mean tempos; however, given the very low tempos, these results were uncertain and could have been due to chance.

Exercises based on the 2:3 framing rhythm showed a higher mean tempo than other groups of exercises, and the slowness of the framing rhythm was noted as difficult for these exercises. The relative speed of the framing rhythm therefore seems to influence the lowest attainable tempos. This claim is supported by the small differences found between density groups, together with the high differentiation between the grouping structures for the 10:3 and 14:3 exercises. This result would also be in line with Handel's (1984) focus on the duration of the interelement intervals, rather than cycle length, as an influencing factor. Framing rhythms slower than the pulse seem to restrict the tempo limits most notably, possibly since the duration between the strokes in the framing rhythm becomes too slow to be clearly grasped. However, the 2^n:3 exercises were the only exercises tested in which the framing rhythm was slower than the pulse. Future research that tests more configurations in which the framing rhythm is slower than the pulse would therefore be of interest.

5.2 Structures

Regarding the polyrhythmic structures, the configurations and combinations affected which performance strategies could be used.[49] The strategies were generally tied to a certain tempo range, outside of which the exercises were either unstable or the mental organisation

[49] The performance strategies are described in section 2.4.

could not be sustained. Motion-based strategies were generally helpful in all tempos; a slower sense of pulse was helpful in high tempos; and subdivisions could stabilise performance in low tempos but were not helpful for more complex exercises.

5.2.1 Motion-Based Strategies and Timing Goals

For both high and low tempos, the clear timing goals and motion-based strategies made performance more stable. Given the close connection of rhythm and motion (Bengtsson, 1987, p. 71), this result was not surprising. For the 7:1 exercises, focusing on the right-to-left motion in the downbeats was helpful in note rates around 600bpm and higher, corresponding to the perceptual threshold of fast tempos (see section 1.2.2). For the 1:1–5:1 exercise, focusing on the right-hand motion worked well in high tempos, as it created longer lines and a slower sense of pulse. Similarly, grouping the 3:3–2:3 exercise into a two-cycle tactus (six pulses), instead of a one-cycle tactus, allowed more relaxed playing and higher tempos to be attained. These findings are congruent with the notions that notes in fast tempos are timed in groups of notes, and that a slower sense of pulse facilitates fast playing (Bogacz, 2005, p. 33; Clarke, 1999, p. 495; Mclaughlin & Boals, 2010).

Motion-based strategies were especially helpful in slow tempos. Exercises in which the reference was congruent with the framing rhythm were generally easier to play due to the preserved motion in the shift between the rhythms. This effect was most clear when a right-to-left motion could be used in both rhythms of the exercise – that is, in exercises with an uneven embedded subdivision. The same effect was present even if the movement pattern was not physically carried out but only thought mentally (see section 4.3.1). For other exercises, body motions following the pulse could create a sense of continuity and stability in slow tempos.

A common characteristic in these examples is that a clear timing goal facilitated stable performance. The mental organisation and motion strategies were interconnected in similar ways for both slow and fast tempos; for some exercises, the motion created stable timing goals, whereas for other exercises, a stable sense of pulse allowed for more relaxed movements. Nevertheless, there were differences between types of exercises. For the fast tempos, the faster rhythm determined the strategy used to a larger extent, focusing on a relaxed feeling in the played rhythm and the experienced pulse. For the slow tempos, it was rather the combination that influenced which strategies were used, with a focus on continuity in the played rhythm or the sense of pulse.

It is possible that the lack of clear timing goals contributes to performance difficulties in rhythms such as those in Ferneyhough's *Bone Alphabet*. Indeed, Schick's (1994, p. 138) solution for learning the rhythm in Figure 15 involved translating the 6:7 ratio into a change of tempo, thus relying on a clearer sense of pulse to be able to time the rhythm correctly in practice. Future studies could investigate how different means of creating timing goals, such as focusing on the hand motions, creating mental movement patterns, and body movements, can be used in general polyrhythmic training but also in connection to specific pieces. For instrumentalists other than percussionists, the movements involved will be different, and the strategies would likely have to be adapted. For instance, the focus on hand motion could be replaced by a focus on important notes in the structure.

5.2.2 Pulse Tactus and Multiple-Pulse Tactus

In fast tempo exercises, I tended to spontaneously group the metronomic pulses into a tactus of two- or three-beats in pulse tempos around 150bpm. For all X:4 exercises, pulses were grouped into a cycle level tactus in tempos above 260bpm. In the tempo span around 100–140bpm, there was a transitional range in which it was possible to choose whether to feel the tactus on pulse level or with a multiple-pulse tactus. Similarly, there was a transitional range for the X:4 rhythms from a two-beat to a four-beat tactus in the tempo span around 180–260bpm. Divided in half (i.e. 90–130bpm), these tempos were rather congruent with the 100–140bpm transitional range. Often, both a single- and a multiple-pulse tactus proved effective for some part of the transitional range, but there was a general tendency that more complex polyrhythms, especially combined together (e.g. 5:2–7:2), were more stable with a slower tactus. In these exercises I often grouped the pulses in the lower part of the transitional range.

For the slow polyrhythmic exercises, the tactus shifted from a cycle-level tactus to a pulse-level tactus in the general tempo range of 110–90bpm. Although these tempos correspond to the transitional range mentioned earlier, the pulse tempos are on the low side. As in the fast tempo exercises, the more complex rhythms (primarily the references in the X^n:3 set) were felt with a pulse tactus in lower tempos.

The lower tempo transitions for the slow tempo exercises could possibly be explained by the more elaborate rhythms in the X^n:2 and X^n:3 set. Since the more complex fast tempo exercises were felt with a multiple-pulse tactus in lower tempos than simpler exercises, there seems to be some support for this claim. However, if increased complexity correlated to the tempo range in which a pulse-level or a cycle-level was most effective, the results of the exercises should be evenly distributed over the transitional range, in accordance with the complexity of the exercises. This was not found to be the case. Instead, there were two pronounced peaks around 150bpm for fast tempo exercises and 100bpm for slow tempo exercises. These peaks likely reflect the way I took notes for the fast and slow tempo exercises. By going from slow to fast, the pulse tactus was noted first, and only when it was no longer possible to sustain this feeling of pulse, the shift to a cycle-level tactus was noted. When starting from a high tempo, it was usually easier to feel the exercises on cycle level, if possible, and the shift was noted when it was noticeably easier to feel the rhythms on pulse level.

For the polyrhythmic exercises, a tactus felt over multiple pulses corresponded to a beat-division strategy, whereas a pulse-level tactus corresponded to a composite-pattern strategy. Since the changes of tactus stayed rather consistently around pulse tempos of 150bpm and 100bpm for the fast and slow tempo exercises respectively, the distinction between the strategies seems most closely related to pulse tempo. For the fast tempo exercises, the tempo of the tactus shift corresponded to Fraisse's *temps courts* and *temps longes* (as cited in Clarke, 1999, pp. 474–475), but the lower transitions found in the slow tempo exercises do not fit with this explanation. The differentiated tempos of the tactus shifts were only found in polyrhythmic configurations, reflecting the possibility of a choice between a single- or a multiple-pulse tactus in the transitional range.

5.2.4 Subdivision Strategies

For all exercises without an embedded subdivision, subdivision strategies were helpful in low tempos. Generally, mental subdivisions were used below 60bpm for simple configurations, down to around 35bpm for more complex configurations. When a noncongruent reference was used in the X^n:3 set, I sometimes experienced that the subdivision disturbed the timing while shifting between the rhythms. Therefore, for the X^n:3 set, subdivision was less frequently used, mainly occurring in pulse- or frame-condition references. For some of the simpler exercises, it was possible to use a common subdivision for both rhythms. More often, however, I subdivided either both or only one rhythm to match the pulse.

In the lower tempo ranges, the polyrhythms could be played with a composite-pattern strategy or with subdivision strategies. The choice of strategy had distinct effects on timing. The composite-pattern strategy relied on an internalised sense of the polyrhythmic configuration as a whole. With this strategy my timing was primarily guided by the relation between the played rhythm and the pulse and focused on the framing rhythm. If embedded subdivisions were used in the composite-pattern strategy, they were timed in relation to the framing rhythm much as subdivisions are timed to a pulse. The pulse took on a background function but was still a relevant part of the pattern. In very slow tempos, however, the pulse was primarily used as a checkpoint for the timing of the played rhythm. Instead of anticipating the pulse, I reacted to the pulse as early, late, or in time in relation to the internalised sense of the rhythm. Therefore, slow tempos were often played with considerable rubato and could be difficult to time well.

The subdivision strategy focused on connecting the pulse and the played rhythm by a common subdivision. The pulse and the subdivision were often most prominent, and the played rhythm had a syncopated on- or off-beat feeling. In the few cases when this strategy was used for exercises with embedded subdivisions (e.g. 3^3:2, $2^4/4^2$:3), the hierarchy of the framing rhythm and the embedded subdivision disappeared, and all strokes were played rather equally. As the subdivision needed to be clearly counted, this strategy was easier in slower tempos and often proved playable well below 30bpm.

I found, however, two different subdivision strategies for certain configurations. For the 2:2–3^3:2 exercise, there were two distinctive strategies. In one strategy, I subdivided the pulse into triplets and played the 3^3:2 rhythm as a 3:2 composite pattern superimposed over the triplet subdivision. In the other strategy, I subdivided the pulse into nonuplets and timed all played notes in relation to this subdivision. Because of the superimposed 3:2 pattern, the first strategy still somewhat resembled the composite-pattern strategy, whereas the second was a more typical subdivision strategy. The first strategy was easiest to play in tempos around 40bpm but became increasingly unstable in lower tempos, therefore resembling the composite-pattern strategy in regard to the tempo range in which it could be used.

The timing differences found between the composite-pattern strategy and the subdivision strategies are likely related to the control of the timing as either connected to a fast or a slow level in the metrical hierarchy. In the subdivision strategy, the timing is controlled on a faster level than the played rhythm, leaving less room for flexible timing. When the timing is controlled on a slower level, as in the composite pattern, the faster notes are more flexible (compare Clarke, 1999, p. 495; Kvifte, 2007, p. 70).

5.2.5 Tempo Ranges of the Performance Strategies

Following is a short summary of the conclusions regarding performance strategies in relation to tempo ranges.

Motion-based strategies were used in all tempo ranges, but in different ways depending on the configurations and the tempos involved.

Since the beat-division strategy required the pulses to be grouped into a multiple-pulse tactus, the beat-division strategy was usable in a tempo range from pulse tempos around 100bpm (under which tempo a pulse-level tactus was always used) to 570bpm, when not limited by note rate or other factors. Since the strategy is felt over a single experienced pulse, it has an obvious connection to beat-division polyrhythms.

The composite-pattern strategy was used when the tactus was not felt on cycle level, restricting the experienced pulse tempo to 150bpm at the highest.[50] The lower tempo limit is restricted by stability of performance without subdivision for assistance. The composite-pattern strategy has a clear connection to time-span division polyrhythms; the tempo range of time-span division polyrhythms could be considered limited to the tempo in which the composite-pattern strategy can be used.

The subdivision strategy proved most effective in tempos in which the subdivision was easily counted; both the pulse tempo and the note rate of the common subdivision affected the tempo range in which the subdivision strategy is helpful. Therefore, the strategy was not often used for more complex exercises. Since this strategy relies on a common subdivision, it has a clear connection to phrase length polyrhythms.

[50] For the X:4 rhythms, the tactus could be felt on half-measure level, thereby creating the experience of a X:2 rhythm. The pulse tempos in which the composite-pattern strategy could be used was therefore practically doubled. Nevertheless, the *experienced* pulse tempo in the X:4 rhythms was not higher than 150bpm. I expect that X:6 polyrhythms can be structured in similar ways.

These results show that strategies are tied to specific tempo ranges in congruence with the implications in the literature (e.g. Bogacz, 2005; Clarke, 1999, p. 495; Handel, 1984; Mclaughlin & Boals, 2010). Even if there was some overlap in the tempo ranges, the strategies had distinct effects on timing and stability. Since the beat-division, composite-pattern, and subdivision strategies correlate to the three main polyrhythmic types (see section 2.1),[51] these findings ground the theoretical distinction between the polyrhythmic types in practical experience.

Organising the performance strategies in this order gives an impression of continuity: if a configuration is played in sufficiently high tempos, it can be considered a beat-division polyrhythm, and in sufficiently slow tempos, it can be considered a phrase length polyrhythm. This continuity might seem obvious, but it only applies when the same polyrhythmic configuration can belong to all types; some configurations are only possible in certain polyrhythmic types. For instance, the syncopated patterns in Sandström's *Drums* can only be constructed as a phrase-length polyrhythm, and the nested rhythms in Ferneyhough's *Bone Alphabet* are only conceivable as time-span division polyrhythms. Furthermore, even if a configuration can be structured as any polyrhythmic type, this does not mean that the playable tempo ranges of the different structures always overlap.

These distinctions point to the importance of considering polyrhythms in a multifaceted way. When performing or learning to play polyrhythms, an understanding of the configuration, grouping structure, and intended tempo should inform the choice of performance strategies.

5.3 Other Considerations

5.3.1 The Different Conditions of Fast and Slow Tempos

Regarding both tempo limits and the performance strategies used, fast and slow tempos involved different factors that influenced playability. For the fast tempos, the combination of exercises determined the tempo limit to a large extent, but the performance strategies were mostly related to the faster rhythm in the exercise. In slow tempos, the framing rhythm was perhaps most influential on tempo limits, but the combination of rhythms was highly influential on the types of strategies that could be used. Furthermore,

[51] The extrametrical polyrhythms are not considered a main type of polyrhythm due to their ornamental character.

the fast tempo exercises were often limited by personal physical constraints. The slow tempo exercises, however, were clearly perceptually limited. These observations are summarised in Table 22.

Table 22

Factors influencing playability in high and low tempos.

	High tempos	**Low tempos**
Strategies	Single rhythms	Combinations of rhythms
Tempo limits	Combinations of rhythms	Single rhythms
Constraints	Physical	Mental

These results show that different approaches are needed depending on the tempo range and whether mental strategies or performance limits are to be tested, holding implications for future studies. Regarding the fast tempo limits, the exclusion of non-polyrhythmic exercises made comparisons of exercises across different sets difficult. To be able to compare exercises across different sets, the exercises should include the same played rhythms over cycles with various number of pulses (i.e. $3{:}Y_a$–$4{:}Y_a$ and $3{:}Y_b$–$4{:}Y_b$). In a pedagogical situation, polyrhythms intended to be played in a low tempo should be practiced in a musical context, in order to avoid reliance on a single strategy that might be inefficient in other situations.

5.3.2 Comfortable Tempo Ranges

For many exercises, I noticed that the rhythms were comfortable to play in certain tempos but unstable and stiff in others. For most exercises, performance was unstable in tempos below 40bpm, supporting the choice of 40bpm as a limit for practically usable tempos. Otherwise, the comfortable and stable tempo ranges varied across exercises. Several factors that could potentially have influenced my perception of the comfortable tempo range will be discussed in the following section.

Regarding the relation between complexity and performance accuracy, Grieshaber (1990, pp. 179–180) proposed that either the number of elements or the length of the rhythm could be measurements of complexity related to performance accuracy. My study found little evidence to support this claim, as most exercises could be comfortably played in some tempo range. However, performance accuracy was not compared across exercises, so conclusions about performance accuracy between exercises cannot be drawn.

A few alternative explanations regarding factors that may influence the comfortable playing range – correlated with stability of playing – are discussed below.

In the X:2 set, exercises in which no rhythm matched the pulse were more stable when a cycle-level tactus was comfortable. Yet, I also noted that the 5:2–7:2 exercise was comfortable at 80–70bpm and stable down to 45bpm. Similarly, the 3:3–2:3 exercise was uncomfortable in the tempo range of 300–360 (tactus 100–120bpm) but could be played comfortably in higher and lower tempos. Thus, middle-range tempos were not necessarily the easiest to play; in other words, difficulty did not necessarily increase linearly towards the extreme tempos. For the X:2 exercises without pulse-matching rhythms, the ambiguous range of 100–140bpm seems to be correlated with unstable performance, perhaps since neither the pulse nor the cycle could be easily perceived as tactus.

In the X^n:3 set, performance was more stable when the framing rhythm was in a comfortable tempo range. For the $2^4/4^2$:3 exercises (section 4.5.2), the slower 2:3 framing rhythm was more relaxed and stable in tempos around 130–100. In still lower tempos, the slowness of the 2:3 framing rhythm was instead destabilising, but the faster 4:3 structure was more effective. In tempos lower than 50bpm, the 2^4:3 structure was too slow to sustain mentally, and only the 4^2:3 structure could be used. This explanation fits well with the observations from the 3:3–5^4:3 exercise (section 4.5.1), in which my timing was primarily guided by the framing rhythm. One could hypothesise that a polyrhythm can be comfortably played when the framing rhythm is within a medium tempo range; a slower framing rhythm should be more effective in medium-high tempos, whereas a faster framing rhythm is preferable in slow tempos.

Another possible explanation for the comfortable playing range relies on the timing relation of the pulse and the played rhythm. The minimal duration of this timing relation corresponds to the implied subdivision of the polyrhythm. As the tempo slows down, this duration widens; when the duration can be clearly discriminated, the rhythms can be comfortably played. Although I did not thoroughly investigate this phenomenon, I noted a few examples when I experienced this timing relation clearly, mainly in the 2^5:3 and 5^2:3

exercises, as well as exercises with a 7:3 reference. The found tempo range of the minimal timing relation was rather wide, ranging from 500 to 350bpm (120 to 170ms). This finding may reflect a range from noticeable difference to clear difference, but it also may due to my unsystematic treatment of the issue.

However, when comparing my notes regarding comfortable tempos for the exercises with a 7:3 reference, the results are not similar. In 90bpm (faster than the 120–170ms span), two exercises were notably uncomfortable, in line with the timing relation explanation, but five exercises could be played comfortably.[52] Thus, the timing relation explanation is probably not relevant for the comfortable playing range but could still be helpful for learning polyrhythms, as a clearly felt pattern should be easier to internalise.

It is also possible that note rates within a certain range influenced comfortable playing. For some exercises in the X^n:3 set, I explicitly noted the "most comfortable tempo". Although the notion and its criteria of judgement were undefined and quite subjective, I found a strong tendency that note rates around 458 to 480bpm were comfortable to play. Eight out of the twenty-nine noted exercises were comfortable outside of that range, but only two were comfortable considerably outside of that range.[53] For most exercises, I notated a span of tempos, in which the lower note rates in the span were generally between 300 and 373bpm. This tempo span might provide a good starting point for practicing polyrhythms with embedded subdivisions. However, for the few exercises in the X:3 set in which I noted a comfortable playing range, the note rates fell between 210 and 116bpm, indicating that more factors than simply note rates influence the comfortable playing range.

Future research could more thoroughly examine the comfortable tempo ranges. Since this issue regards stability in performance, timing deviations qualified by the performer's experiences may prove to be a good measurement for testing the comfortable tempo ranges. Based on the previous discussion, I propose four hypotheses for factors influencing the stability of performance:

[52] Uncomfortable exercises: 7:3–2^4:3 and 7:3–5^4:3. Comfortable exercises: 7:3–2^5:3, 7:3–2^7:3, 7:3–4^5:3, 7:3–5^2:3, 7:3–7^2:3.

[53] The dataset for the examined exercises is included in Appendix 1, Table L. The eight exercises outside of the common range were the four 7^2:3 exercises (420bpm), 2:3–2^4/4^2:3 (320bpm), 5:3–2^5:3 (433–300bpm), 7:3–2^5:3 (333bpm) and 3:3–5^2:3 (433–300bpm). Common to all of them is a low density.

- Performance is unstable when neither the pulse nor the cycle can be clearly perceived as tactus.
- Performance is stable when the framing rhythm is in within a medium tempo range.
- Performance is more stable when the timing relation between the played rhythm and the pulse can be clearly felt.
- Performance is stable when the note rate of the played rhythm is within a certain range.

The first hypothesis could be tested through comparing the stability of polyrhythmic configurations across different tempos. Because all tested exercises were easily grouped in either a two- or three-beat tactus, five-beat polyrhythms (which do not allow an even grouping of the pulses) would be of special interest. Since the X:2 exercises from which I derive the first hypothesis were only unstable when neither rhythm in the exercise matched the pulse, the potential effect of the interaction between rhythms should be considered. The second hypothesis could be tested by comparing the stability of polyrhythms with the same density but with different grouping structures across a range of tempos. The influence of the level of differentiation between the grouping structures could also be examined. The third hypothesis could be tested by comparing the stability of configurations with varying density across different tempos. The tempo should be adjusted so that the timing relation is the same for the compared configurations. The fourth hypothesis could be tested like the third, but with the tempos adjusted so that the note rate is the same for the compared configurations.

5.3.3 Tempo Limits and Performance Difficulties

I proposed that the tempo limits, reflecting performance difficulties, could be taken as an indication of complexity: a rhythm playable in a narrower tempo range could be considered more difficult and therefore more complex. However, both the 2:3–4:3 and 3:4–5:4 exercises were notably more difficult than the other exercises. While the 2:3–4:3 exercise was playable in substantially lower tempos than the 2:1–4:1 exercise, the 3:4–5:4 exercise was playable in comparable tempos to the 3:1–5:1 exercise when it was replayed. In addition, this proposition assumed that polyrhythms were most comfortable to play in a middle-range tempo, becoming gradually more difficult towards extreme tempos. The results did not support this proposition, as the exercises were often more stable in relatively high and low tempos. Equating tempo limits with performance difficulties is therefore problematic.

Regarding the tempo limits, the effects of adjacency and differentiation could be compared to the maximum tempo limit for a continuous stream of notes. Since the highest playable tempo varies across individuals (see section 1.5.4), more proficient and less proficient performers could be allocated to different groups. Supposing there is a highest physically playable tempo for all individuals (for example, 1200bpm), the effect of differentiation should be less marked for technically proficient performers, who can play continuous streams of notes in higher tempos. One could also examine whether the effect of adjacency is connected to technical proficiency (proficient performers attaining higher tempos in adjacent-rhythm exercises) or if this effect is more "hard-wired" (no significant difference found across participants).

Performance difficulties are perhaps best described in terms of stability. Timing deviations are, therefore, a preferable measurement for situations in which performance difficulties are to be tested. However, as timing patterns vary for different tempos (see section 1.5.3), it is not possible to claim that larger timing deviations found in separate tempo ranges for the same configuration necessarily correspond to performance difficulties. Moreover, since the stable tempo ranges varied across exercises, the relative stability of different configurations cannot be compared in a single tempo. Therefore, a combined approach might be justified. A polyrhythm (for example, 9:2) can be compared to a pulse-matching rhythm (for example, 9:3) with the same note rate. This would then allow the comparison of timing variations for different pulse conditions, with reference to performed timing rather than a mechanical norm. If the same procedure is made in various tempos, the consistency of these variations across the tempo range could be examined.

5.4 Conclusion

This thesis aimed to explore the performance possibilities of single-line polyrhythms in the context of percussion performance, focusing on how polyrhythmic configurations relate to tempo limits, and how tempo influences the polyrhythmic structure. Besides its purpose in this thesis, the design of the practical study itself opens possibilities for exploring promising fields of further investigation, such as the effects on tempo limits of adjacency and differentiation.

While the study was designed for the purpose of investigating tempo limits and structures in single-line polyrhythms played on a snare drum, the exercises were also written as musical exercises, which I experienced as useful as a tool for learning polyrhythms. The fast tempo exercises acted as good technique builders, and the slow tempo exercises strengthened my sense of timing. The progression of the exercises was also helpful, as the simpler exercises prepared for the more elaborate exercises. Elements from the design of the practical study are therefore likely adaptable to studies with pedagogical aims. Informed by the knowledge gained form this study, more focused studies can be designed. In the following section, a few suggestions for future studies are presented.

To extend the results of this thesis and further explore performance limits, a follow-up study involving more percussionists could be conducted. This study could, for instance, include the tests on tempo limits and stability mentioned in section 5.3.3 and could test specific exercises.[54]

Similar studies could also be conducted on other instruments. However, due to the connection between physical motion and tempo limits, the proposed effects may not prove relevant for other instruments. An explorative approach on other instruments would therefore be beneficial. Such a study could adapt the design of the present study to the specific instruments involved. Potential issues could be the choice of notes and scales to play, and for some instruments the choice of register would need consideration.

Some suggestions for adaptations of polyrhythmic practice to other instruments are found in the pedagogical literature. Magadini (1993) proposes the use of scales and melodies while practicing. Pianist Graham Fitch adds an important aspect to the method – using single notes as an anchor for the faster rhythms (Informance, 2019). In other words, if a scale is played, one starts by playing only the notes falling on the rhythmically accented parts of the scale. While Fitch only plays these accented notes on the beat, there is nothing preventing using the same principle for time-span division polyrhythms. In this approach, the accented beats are practiced like the framing rhythm, and the faster notes in the scale are later added as an embedded subdivision.

[54] Depending on the aim, various approaches are possible. One approach for testing the effects of differentiation would be to increase the length of the rhythms (e.g. test 1:1–3:1, 1:1–4:1, 1:1–5:1 etc.). Another approach would be to keep the faster rhythm the same and increase the speed of the slower rhythm (e.g. test 1:1–4:1, 2:1–4:1, 3:1–4:1 and similar). One could also test if polyrhythmic configurations influence the results (e.g. test 1:1–5:1, 1:2–5:2, 1:3–5:3 and similar).

Another practical application would be to adapt aspects of my study into a method for practising specific pieces of music.[55] It would then be possible to test the applicability of the ideas behind the design of the exercises with a concert-performance context in mind. Such a study would also be easier to conduct than the other suggestions, since both percussionists and other instrumentalists could participate, and the study would require less prior experience with polyrhythms. Topics of interest include whether the proposed method is effective for learning polyrhythmic passages in music, if the method works for most musicians, and how different instrument groups respond to the method.

Based on the results from the practical study, a generic approach for practicing hierarchical time-span division polyrhythms could be outlined as follows. The practice of basic polyrhythms should start in a tempo range when the tactus can be felt on cycle level, so that the relation between the played rhythm and the pulse is less important. Assuming that a salient timing relation between the played rhythm and the pulse facilitates learning, the rhythm should be practiced down to a pulse tempo in which this timing relation is clear. When the basic polyrhythm has been mastered, the embedded subdivisions can be added. As pointed out, polyrhythms are not a unified concept and not all strategies work for all purposes. Because of this, the exercises would have to be written with the intended piece in mind.[56]

This thesis intended to explore polyrhythmic performance, providing nuance to the understanding of the concept of polyrhythms through practice and providing a foundation for practical applications and future studies. The aim was not to identify fixed limits but to open a field of investigation. The self-observational approach allowed me to have control over the study and gain an in-depth understanding of polyrhythmic performance. The study has shown that polyrhythmic configurations are playable in a wide range of tempos and possible to structure in multiple ways. The structures lend certain characteristics to the

[55] In connection with this line of research, I will have the opportunity to develop this practical approach along with my supervisor, Dr. Laura Gorbe-Ferrer, within the framework of the Aural Skills subject at the Norwegian Academy of Music (academic year 2022/2023). The idea of this new project is to apply knowledge from this thesis to challenges found in the classical repertoire.

[56] For example, to practice the 7:2 rhythms in Cage's *Third Construction* (score referenced in Figure 12, section 2.1.4), the method could be applied in the following manner: the melodic figure is learned, with a metronome click marking the downbeat of the measures. When this can be played comfortably in rather high tempos, the metronome instead articulates two beats per measure. The rhythm is then practiced until playable in a tempo lower than performance tempo. After this, one adds the surrounding rhythmic figures and practice in both higher and lower tempos.

polyrhythmic configurations, specific to certain tempos. Considering polyrhythms as more than an abstract ratio thus provides a means for understanding, learning, and performing polyrhythms. While the focus of this thesis has been on exploring technical aspects of performance, practical applications are the ultimate goal. I hope this thesis can provide a foundation for future studies, informed pedagogical approaches, and musical performance.

References

Musical Scores

Beethoven, L. van. (1863). *Symphony No. 7* [Musical score]. Breitkopf und Härtel. (Original work published 1816)

Cage, J. (1970). *Third construction: 4 percussion players* [Musical score]. Peters Henmar Press.

Chopin, F. (1880). Fantaisie-Impromptu, Op. 66 [Musical score]. In K. Klindworth (Ed.), *Oeuvres complètes de Frédéric Chopin* (Band 1: Impromptus, pp. 149–155). Bote & Bock. (Original work published 1855)

Chopin, F. (ca. 1905). *Nocturnes* (R. Pugno, Ed.) [Musical score]. Universal Edition. (Original work published 1832)

Ferneyhough, B. (1991). *Bone alphabet: For a percussionist* [Musical score]. Peters.

Ligeti, G. (1986). *Études pour piano: Premier livre* (Vol. 1) [Musical score]. Schott.

Rossini, G. (1994). William Tell Overture [Musical score]. In *William Tell and Other Overtures in Full Score* (pp. 1–54). Dover Publications. (Original work published 1829)

Sandström, S.-D. (1984). *Drums: For percussion quintet* [Musical score]. Nordiska musikförlaget.

Stout, G. (1977). *Two Mexican dances for marimba* [Musical score]. Alfred.

Takemitsu, T. (1981). *Rain tree: For 3 percussion players* [Musical score]. Schott.

Xenakis, I. (1970). *Persephassa: Pour six percussionistes*. Edition Salabert.

Videos

ArtOfDrumming. (2019, August 30). *Introduction to Moeller technique with Claus Hessler* [Video]. YouTube. https://www.youtube.com/watch?v=NF5C3ZNA_Vk

David Bruce Composer. (2019, March 4). *100 year old polyrhythms vs new polyrhythms* [Video]. YouTube. https://www.youtube.com/watch?v=svbPKJBJz8I

DRUMMERWORLD. (2020, March 6). *Jim Chapin drum clinic: The Moeller technique—Part 1 [Video]. YouTube*. https://www.youtube.com/watch?v=yaRl3IbMjgc

Informance. (2019, October 16). *Chopin—Fantaisie Impromptu—How to manage the 4:3 polyrhythm* [Video]. YouTube. https://www.youtube.com/watch?v=U5itLZaqYx8

Pianist Magazine. (2016, March 14). Piano lesson on polyrhythms [Video]. YouTube. https://www.youtube.com/watch?v=fH6XDUD1swE

Shawn Crowder. (2020, June 23). *How to Play Nested Tuplets (polyriddim explained)* [Video]. YouTube. https://www.youtube.com/watch?v=xv05y31U1p4

Signals Music Studio. (2018, September 1). *POLYMETER - Understanding and Using Complex Rhythms [MUSIC THEORY - RHYTHM]* [Video]. YouTube. https://www.youtube.com/watch?v=MqyrPzvN3Zc

Literature

Agawu, K. (2003). *Representing African music: Postcolonial notes, queries, positions*. Routledge. https://doi.org/10.4324/9781315810942

Beauvillain, C., & Fraisse, P. (1984). On the temporal control of polyrhythmic performance. *Music Perception*, *1*(4), 485–499. https://doi.org/10.2307/40285274

Bengtsson, I. (1987). Notation, motion and perception: Some aspects of musical rhythm. In A. Gabrielsson (Ed.), *Action and perception in rhythm and music* (Vol. 55, pp. 69–80). Royal Swedish Academy of Music Stockholm.

Bogacz, S. (2005). Understanding how speed affects performance of polyrhythms: Transferring control as speed increases. *Journal of Motor Behavior*, *37*(1), 21–34. https://doi.org/10.3200/JMBR.37.1.21-34

Borgdorff, H. (2006). The debate on research in the arts. *Dutch Journal of Music Theory*, *12*(1), 1–17.

Bortz, G. (2003). *Rhythm in the music of Brian Ferneyhough, Michael Finnissy, and Arthur Kampela: A guide for performers* [Doctoral dissertation, The City University of New York]. UMI.

Brubaker, B., Delaere, M., London, J., Pace, I., Crispin, D., & Decroupet, P. (2009). *Unfolding time: Studies in temporality in twentieth-century music* (Vol. 8). University Press.

Burke, R., & Onsman, A. (2017). Discordant methodologies: Prioritizing performance in artistic research in music. In R. Burke & A. Onsman (Eds.), *Perspectives on artistic research in music* (pp. 3–17). Lexington Books.

Chaffee, G. (1976). *Rhythm & meter patterns: Contains odd rhythms, mixed meters, metric modulation, sticking patterns*. GC music.

Clarke, E. F. (1999). Rhythm and timing in music. In D. Deutsch (Ed.), *The psychology of music* (2nd ed., pp. 473–500). Academic Press. https://doi.org/10.1016/B978-012213564-4/50014-7

Cook, N. (1999). Analysing performance and performing analysis. In N. Cook & M. Everist (Eds.), *Rethinking music* (pp. 239–261). University Press.

Coulembier, K. (2012). Elliot Carter's structural polyrhythms in the 1970s: 'A mirror on which to dwell'. *Tempo*, *66*(261), 12–25. https://doi.org/10.1017/S004029821200023X

Cowell, H. (1996). *New musical resources* (D. Nicholls, Ed.). University Press. (Original work published 1930).

Doğantan-Dack, M. (2015). The role of the musical instrument in performance as research: The piano as a research tool. In M. Doğantan-Dack (Ed.), *Artistic practice as research in music: Theory, criticism, practice* (pp. 169–202). Ashgate.

Dowling, W. J. (1986). *Music cognition*. Academic Press.

Fröjd, V. (2019). *Angående rytm*. https://gupea.ub.gu.se/handle/2077/61944

Gabrielsson, A. (1993). The complexities of rhythm. In T. J. Tighe & W. J. Dowling (Eds.), *Psychology and music: The understanding of melody and rhythm* (pp. 93–120). L. Erlbaum.

Gabrielsson, A. (1999). The performance of music. In D. Deutsch (Ed.), *The psychology of music* (2nd ed., pp. 501–602). Academic Press. https://doi.org/10.1016/B978-012213564-4/50015-9

Gann, K. (1995). *The music of Conlon Nancarrow*. University Press.

Grieshaber, K. (1990). *Polymetric performance by musicians* [Doctoral dissertation, University of Washington]. https://cedar.wwu.edu/sterling_collection/3

Griffiths, P. (1985). *Olivier Messiaen and the music of time*. Faber and Faber.

Handel, S. (1984). Using polyrhythms to study rhythm. *Music Perception*, *1*(4), 465–484. https://doi.org/10.2307/40285273

Handel, S. (1989). *Listening: An introduction to the perception of auditory events*. MIT Press.

Hasty, C. F. (1997). *Meter as rhythm*. Oxford University Press.

Holmqvist, M. (2010). *The reference rhythm method: An intuitive approach to advanced rhythms*. Mulle Holmqvist.

Jersild, J. (1975). *Polyrytmik: Videregående rytmestudier*. Wilhelm Hansen.

Klapp, S. T., Nelson, J. M., & Jagacinski, R. J. (1998). Can people tap concurrent bimanual rhythms independently? *Journal of Motor Behavior*, *30*(4), 301–322. https://doi.org/10.1080/00222899809601346

Kvifte, T. (2007). Categories and timing: On the perception of meter. *Ethnomusicology*, *51*(1), 64–84.

Lerdahl, F., & Jackendoff, R. (1983). *A generative theory of tonal music*. MIT Press.

London, J. (2002). Cognitive constraints on metric systems: Some observations and hypotheses. *Music Perception: An Interdisciplinary Journal*, *19*(4), 529–550. https://doi.org/10.1525/mp.2002.19.4.529

London, J. (2012). *Hearing in time: Psychological aspects of musical meter* (2nd ed.). University Press.

Magadini, P. (1993). *Polyrhythms: The musician's guide*. Hal Leonard.

Mclaughlin, P., & Boals, A. (2010). Perceptual organization can affect performance on rhythmic tasks. *Psychology of Music*, *38*(4), 395–401. https://doi.org/10.1177/0305735609351916

Nicholson, T., & Sabat, M. (2018). *Fundamental principles of just intonation and microtonal composition*. https://marsbat.space/pdfs/JI.pdf.

Palmqvist, B.-O. (2006). *The refinement of rhythm* (Vol. 2). Bopac.

Polak, R. (2018). The lower limit for meter in dance drumming from West Africa. *Empirical Musicology Review*, *12*(3–4), 205. https://doi.org/10.18061/emr.v12i3-4.4951

Pople, A. (1998). *Messiaen, Quatuor pour la fin du temps*. University Press.

Poudrier, È., & Repp, B. H. (2013). Can musicians track two different beats simultaneously? *Music Perception*, *30*(4), 369–390. https://doi.org/10.1525/mp.2013.30.4.369

Redgate, C. (2015). Creating new music for a redesigned instrument. In M. Doğantan-Dack (Ed.), *Artistic practice as research in music: Theory, criticism, practice* (pp. 203–218). Ashgate.

Rink, J. (2015). The (f)utility of performance analysis. In M. Doğantan-Dack (Ed.), *Artistic practice as research in music: Theory, criticism, practice* (p. ?????????????). Ashgate.

Rissman, N. (2005). Cycling through polyrhythms. *Journal of Music Theory Pedagogy*, *19*, 11.

Schick, S. (1994). Developing an interpretive context: Learning Brian Ferneyhough's Bone Alphabet. *Perspectives of New Music*, *32*(1), 132–153. https://doi.org/10.2307/833159

Schick, S. (2006). *The percussionist's art: Same bed, different dreams*. University of Rochester Press.

Stene, H. (2016). *'This is not a drum': Towards a post-instrumental practice*. http://hdl.handle.net/11250/2379520

Stuart, G. W. (2009). *A percussionist's practice* [Doctoral dissertation, University of California, San Diego]. https://escholarship.org/uc/item/1kt9v0km

Toussaint, G. T. (2013). *The geometry of musical rhythm: What makes a 'good' rhythm good?* CRC Press.

Williamon, A., Ginsborg, J., Perkins, R., & Waddell, G. (2021). *Performing music research: Methods in music education, psychology, and performance science*. University Press.

Wood, B. K. (2013). South Indian 'solkattu' and Western music pedagogy: Creating new rhythmic perspectives. *Music Educators Journal*, *99*(4), 63–67. https://doi.org/10.1177/0027432113483839